

Motivation

- **Ocean Lagrangian coherent vortices**, by definition, have the ability to carry particles in their interior without exchange with surrounding waters. Due to this characteristic, these vortices efficiently transport water properties (heat, salt, and oxygen) and tracers (oil, larva, and Sargassum algae) across the ocean.
- Haller and Beron-Vera (2013; **HBV13**) developed a method to identify Lagrangian coherent vortices from the velocity field. However, the HBV13 method is computationally expensive, and its usage is limited to a small community familiar with the algorithm.
- **Machine learning** allows for **faster computation** (1s vs. 180s with HBV13) and **user-friendly detection of Lagrangian coherent vortices** across the oceanographic community.

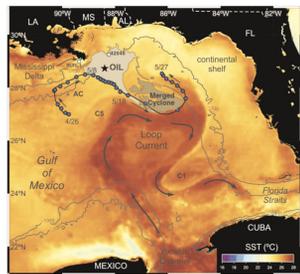
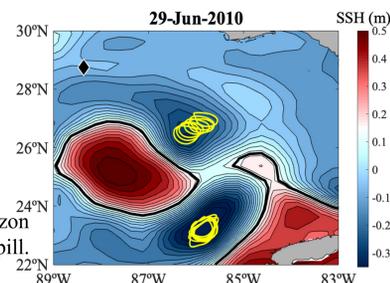


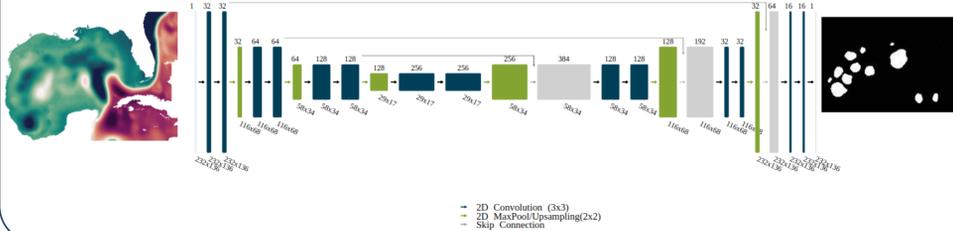
Figure 1. Altimetry sea surface temperature. Walker et al. (2013).

Figure 2. Lagrangian coherent vortex boundaries (yellow contours) during the 2010 Deepwater Horizon (black diamond) oil spill. Hiron et al. (2022)



Machine Learning

- We employ a variation of the U-Net architecture (Ronneberger et al., 2015) to detect the area of Lagrangian coherent vortices from altimetry SSH maps of the Gulf of Mexico.
- The network is trained with contours detected by the HBV13 method for years 2010, 2015, and 2018 and uses the first 60 days of 2020 as the validation set.
- The learning is performed with an Adam optimizer with a learning rate of 0.001 and a scheduler. The learning stops when the validation error does not improve in 100 epochs.
- The loss function is 1 – the Dice Similarity Coefficient, given by $1 - \frac{2A \cap B}{A \cup B}$. A and B represent masks of filled contours of the detected eddies.



Identifying Lagrangian coherent vortices (HBV13)

The methodology requires the evaluation of a set of trajectories distributed across the domain. The evolution of those trajectories can be represented with the flow map $F_{t_0}^{t_0+T}: \mathbf{x}_0 \rightarrow \mathbf{x}(t; t_0, \mathbf{x}_0)$, which maps the final position of a T -long trajectory starting at (\mathbf{x}_0, t_0) .

To identify the structures of a flow field, this Lagrangian method is based on the Cauchy-Green tensor, which is formed from the derivatives of the flow map operator.

$$C_{t_0}^{t_0+T}(\mathbf{x}_0) = \nabla F_{t_0}^{t_0+T}(\mathbf{x}_0) \nabla F_{t_0}^{t_0+T}(\mathbf{x}_0)^T$$

The eigenvectors of $C_{t_0}^{t_0+T}(\mathbf{x}_0)$ represents the stretching *direction* of the flow at \mathbf{x}_0 during the trajectories. Similarly, the eigenvalues of the tensor represents the stretching *magnitude*. In two dimension, the Cauchy-Green tensor is a two-by-two matrix, so it has 2 sets of eigenvectors (ξ_1, ξ_2) and eigenvalues (λ_1, λ_2) defined at each initial position \mathbf{x}_0 .

Lagrangian Coherent vortices are identified as material loops that defy the typical exponential stretching occurring in unsteady fluids. Such loops $r(s)$ are closed trajectories of the vector field η_λ^\pm and *uniformly stretch* by some amount λ . The η_λ^\pm field is formed from a combination of both eigenvectors and eigenvalues of $C_{t_0}^{t_0+T}(\mathbf{x}_0)$, as follows:

$$r'(s) = \eta_\lambda^\pm(r(s)), \quad \eta_\lambda^\pm = \sqrt{\frac{\lambda_2 - \lambda^2}{\lambda_2 - \lambda_1}} \xi_1 \pm \sqrt{\frac{\lambda^2 - \lambda_1}{\lambda_2 - \lambda_1}} \xi_2.$$

The last step of the methodology is to integrate $r'(s)$ and identify outermost limit cycles of η_λ^\pm across the domain. To speed up this process, we used a methodology described in Karrasch et al. (2015), which allows to efficiently identify locations where coherent eddies can be present, hence speeding up calculations.

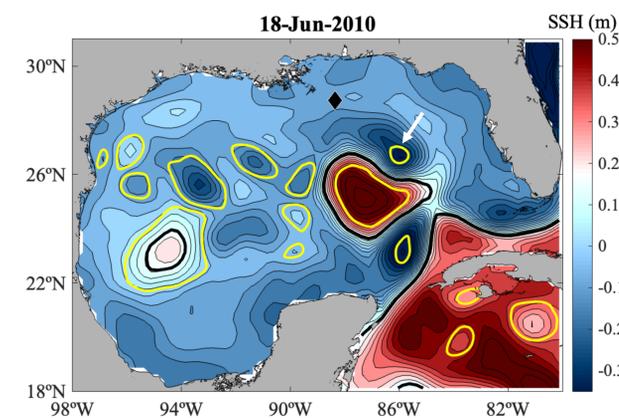


Figure 4. Altimetry Sea Surface Height (SSH; CMEMS Copernicus). The yellow lines show the vortices that remained Lagrangian coherent for 14 days (from 11 June 2010 to 25 June 2010), and the white arrow indicates the vortex responsible for attracting and trapping oil during the 2010 Deepwater Horizon oil spill. The black diamond indicates the location of the Deepwater Horizon oil rig.

Results

Main findings:

- Introducing more data reduces the validation loss more effectively than incorporating additional sea surface height maps into the model. Each training takes approximately 15 minutes on an NVIDIA A100 GPU with preprocessed inputs.
- The proposed technique can successfully identify more than half of the ocean vortices that maintained Lagrangian coherence for fourteen days, relying solely on current SSH information, which is not possible with the HBV13 method.
- The machine learning approach identifies vortices significantly faster, achieving speedups of up to two orders of magnitude (1s with ML vs. 180s with a Julia implementation of HBV13).
- This method also demonstrates generalizability across different dynamical states within the Gulf of Mexico.

Application: Oil transport (e.g., 2010 Deepwater Horizon oil spill), Sargassum and larva transport, and fisheries.

Future work: The model will be trained using all altimetry data available from 1992-2022 (20% will be left out for validation)

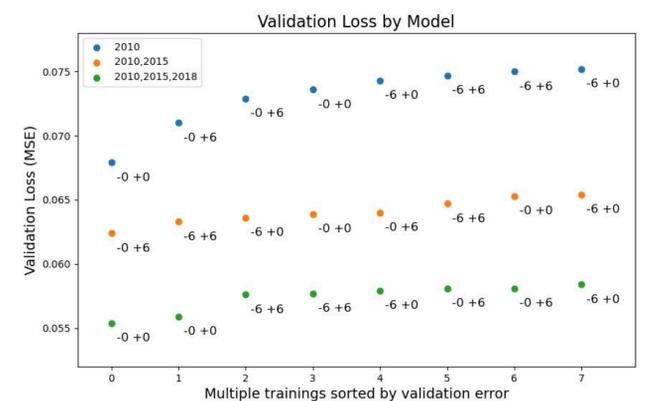
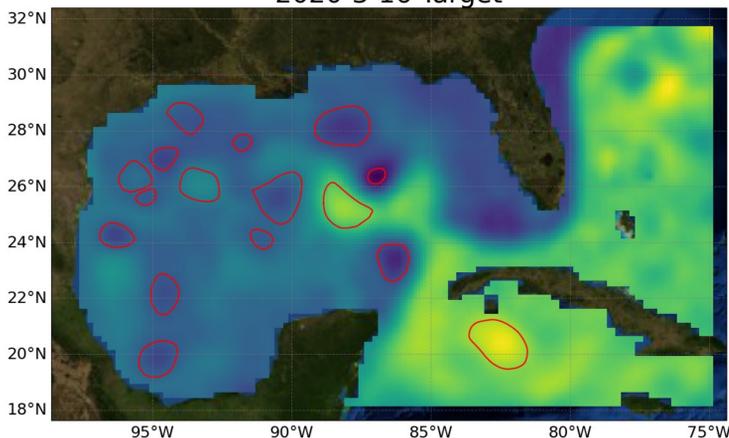


Figure 5. Validation loss for multiple trainings using different input data. The colors indicate the three sets of experiments.

2020-3-16 Target



2020-3-16 Prediction

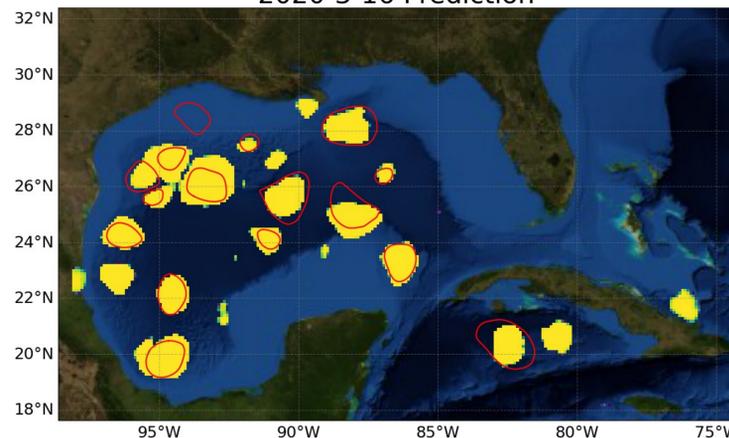


Figure 6. (Left) Altimetry sea surface height and vortex boundaries with 14 days of Lagrangian coherence (red). (Right) Mask of the vortices detected using the U-Net machine learning model trained with years 2010, 2015, and 2018, superposed with the solution from HBV13 (red).

References

- Haller, G., and F.J. Beron-Vera, 2013: Coherent Lagrangian vortices: The black holes of turbulence. *Journal of Fluid Mechanics*, 731, R4
- Hiron L., P. Miron, L.K. Shay, W.E. Johns, E.P. Chassignet, and A. Bozec, 2022: Lagrangian coherence and source of water of Loop Current Frontal Eddies in the Gulf of Mexico. *Progress in Oceanography*. pp. 102876, ISSN 0079-6611.
- Karrasch, D., Huhn, F., & Haller, G., 2015: Automated detection of coherent Lagrangian vortices in two-dimensional unsteady flows. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 471(2173), 20140639.
- Ronneberger, O., Fischer, P., Brox, T., 2015: U-Net: Convolutional Networks for Biomedical Image Segmentation. In: Navab, N., Hornegger, J., Wells, W., Frangi, A. (eds) *Medical Image Computing and Computer-Assisted Intervention – MICCAI 2015*. MICCAI 2015. Lecture Notes in Computer Science, vol 9351. Springer, Cham.
- Walker, N. D., Pilley, C. T., Raghunathan, V. V., D'Sa, E. J., Leben, R. R., et al., 2013: Impacts Of Loop Current Frontal Cyclonic Eddies And Wind Forcing On The 2010 Gulf Of Mexico Oil Spill. In *Monitoring and Modeling the Deepwater Horizon Oil Spill: A Record-Breaking Enterprise*.