A dynamical geography of the Gulf of Mexico

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BIRS on Thursday, January 19

CARTHE CONSORTIUM for ADVANCED RESEARCH on TRANSPORT of HYDROCARBON in the ENVIRONMENT

GULF OF MEXICO RESEARCH INITIATIVE

Objectives

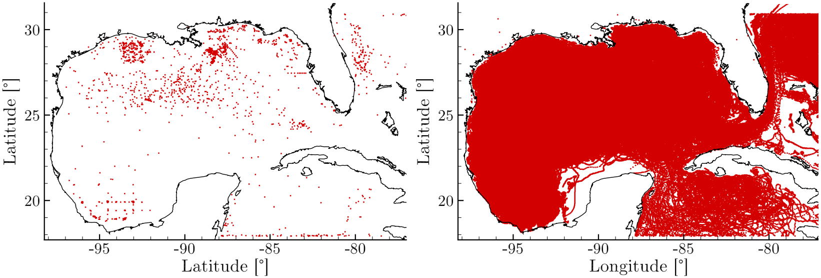
Using **only** drifters data (trajectories) in the Gulf of Mexico (GoM):

- Subdivide the GoM into regions with similar dynamics
- Identify sink and source (attractor and basins of attraction)
- Predict transport of passive and possibly non-passive tracers

Drifters database in the GoM (1994-2016)

Problematics: non uniform data (data per year, type of drifters, time resolution)

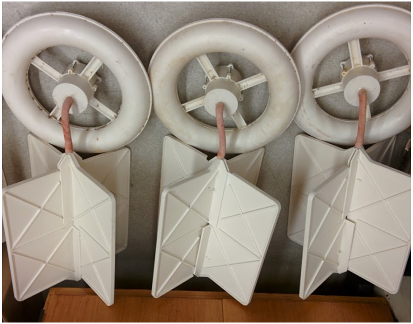
- Left: initial positions
- Right: all trajectories data points



3312 drifters from different sources (LASER & GLAD / CARTHE, GDP / NOAA, BOEM / SCULP, PEMEX / CICESE)

Biodegradable drifters (Novelli et al., 2017, University of Miami)

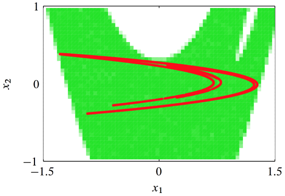
- 1000 drifters deployed during the LAgrangian Submesoscale ExpeRiment (LASER) in 2016
- Total height of 0.6 m
- GPS precision ± 10 m
- Quarter-hourly acquisition ($\sim$ 3 months)



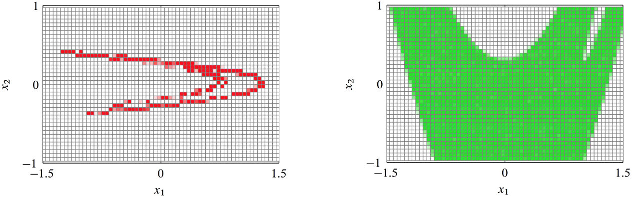
Approximation of an attractor (Dellnitz and Junge, 1997)

Hénon Map: $(x_1, x_2) \to \left(1 - 1.4x_1^2 + 0.3x_1x_2\right)$

10000 iterations

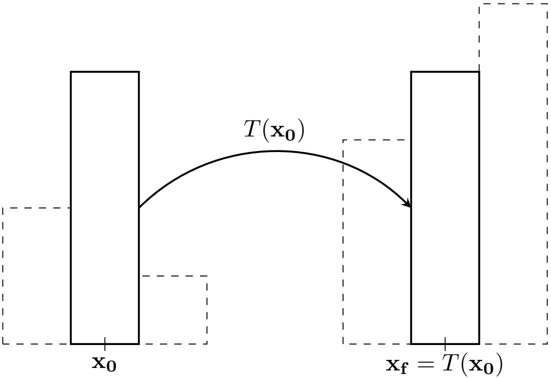


1 iteration



Transfer operator

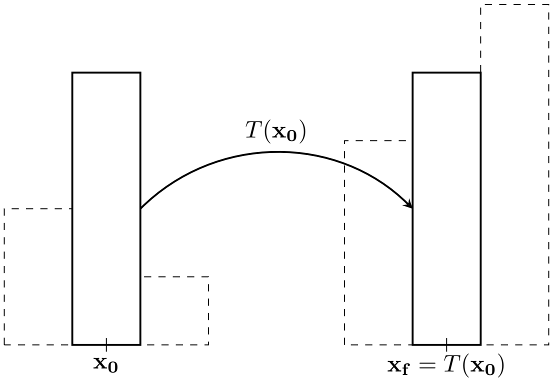
- Domain X
- Map $T : X \circlearrowleft$ acts on a density $f : X \to \mathbb{R}$



Transfer operator

For all the points $\in X$, if the map T is area preserving, the end result can be obtained from the Perron-Frobenius operator ($\mathcal{P}$):

$$\mathcal{P}f(\mathbf{x}) = f \circ T^{-1}(\mathbf{x}).$$



Transition matrix (Froyland, 2001)

- Probabilistic approach known as Ulam's method (Ulam, 1960)
- Subdivision of the domain in boxes $(B_1, \cdots, B_n)$
- From short-time trajectories, we approximate the probability to go from box i to box j:

$$\mathcal{P}_{ij} \approx \frac{\#\{p : p \in B_i \text{ and } T(p) \in B_j\}}{\#\{p \in B_i\}}$$

by counting the number of particles (drifters) in B_i that are mapped into B_j.

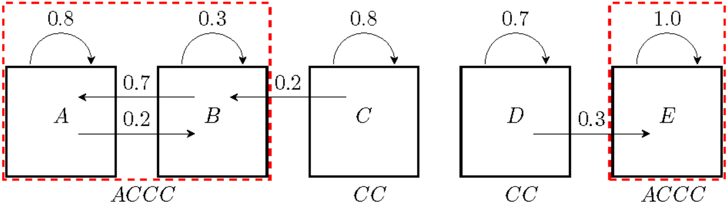
Markov Chain and eigenvectors method (Froyland, Stuart, and van Sebille, 2014)

- $\mathcal{P}$ defines a Markov Chain of the dynamics
- Using initial density $f_0 \rightarrow$ future distribution

$$f_1 = f_0\mathcal{P}$$
$$f_N = f_0\mathcal{P}^N$$

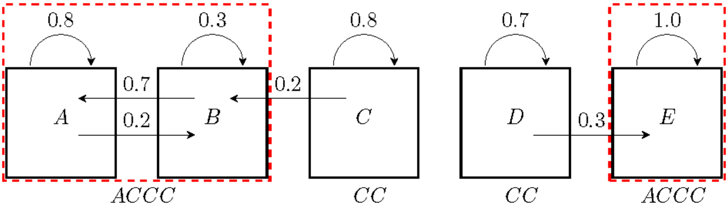
- left eigenvectors ($\lambda L = L\mathcal{P}$)
  - for $\lambda = 1$: invariant distribution
  - for $\lambda \approx 1$: almost invariant distribution
- right eigenvector highlights the basin of attraction

Example with a simple 5 states problem



$$\mathcal{P} = \begin{array}{c|ccccc} & \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \\ \hline \text{A} & 0.8 & 0.2 & 0 & 0 & 0 \\ \text{B} & 0.7 & 0.3 & 0 & 0 & 0 \\ \text{C} & 0 & 0.2 & 0.8 & 0 & 0 \\ \text{D} & 0 & 0 & 0 & 0.7 & 0.3 \\ \text{E} & 0 & 0 & 0 & 0 & 1.0 \end{array}$$

Example with a simple 5 states problem

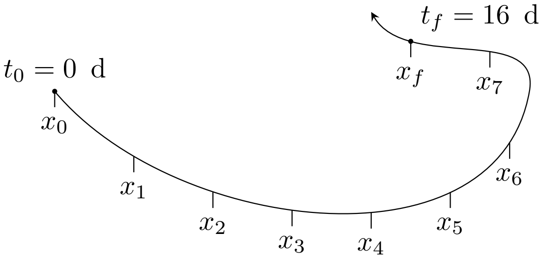


$$L_1^T = \begin{pmatrix} 0.83 \\ 0.55 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad R_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad L_2^T = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad R_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

- L_1: A, B are attractors
- L_2: E is another attractor
- R_1: A, B, C basin of attr.
- R_2: D, E basin of attr.

Hypothesis

All drifters trajectory start at the same time (Autonomous system) and we construct the transition matrix by looking where drifters end up 2 days later (bins size, data).



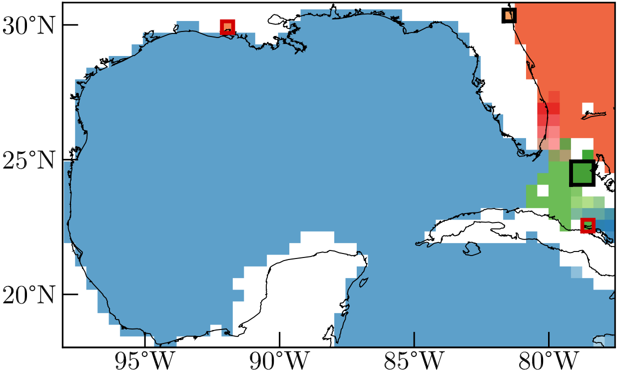
Algorithm

1. Split the domain (GoM) into square boxes
2. For each trajectory segment:
   - find bins i where x_0 is located and store the segment ID in the vector B_i
   - identify bins j where x_f is located and store the segment ID in the vector B_j
3. Calculate the transition matrix $\mathcal{P}_{ij}$ using vectors B_i and B_j
4. Calculate eigenvalues and eigenvectors of $\mathcal{P}$

Strongly connected components (Tarjan algorithm)

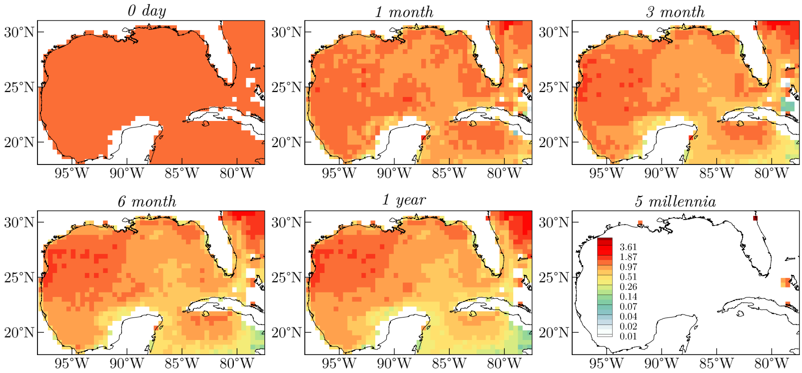
The GoM is almost completely covered by a single CC.

- Borders: Closed Communicating Classes (CCC) in red with Attractive Closed Communicating Classes ($ACCC$) in black
- Handful of drifters and small effect on the general dynamics

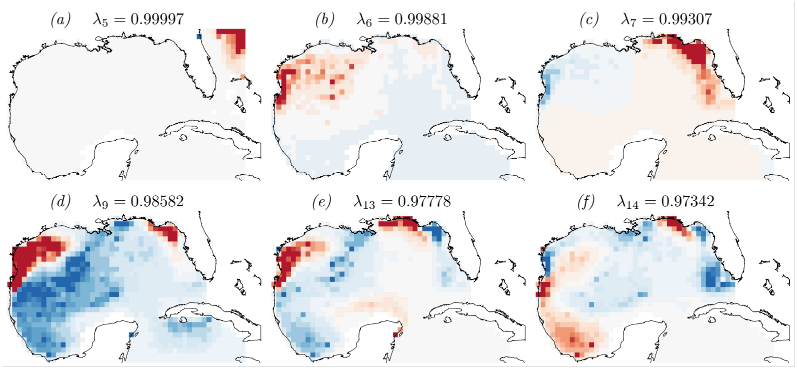


Limiting distribution from a uniform density

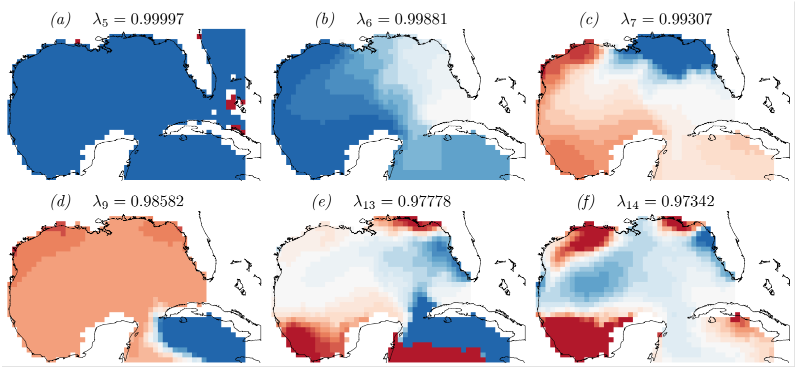
Existence of a westward mean flow: similar to results presented by Sturges, 2016 from SSH data



Top left eigenvectors

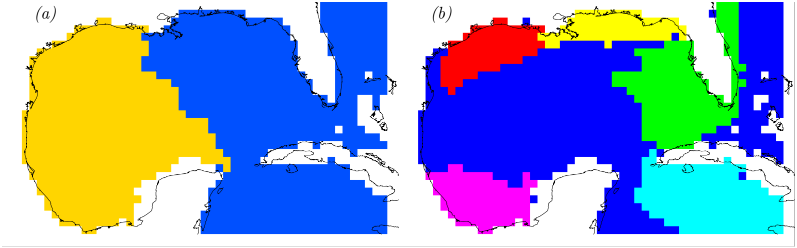


Top right eigenvectors



Dynamical geography

- **left:** the main separation of the GoM
- **right:** the five coastal basins of attraction



Conclusion

- Identified **almost limiting distributions** and corresponding **basins of attraction** from the inspection of the eigenvectors
- Supported by independent observations of a westward mean flow (Sturges, 2016)
- Ability to push-forward a density to "predict" the dispersion (e.g. after an oil spill or to plan a drifters experiment)

Thank you!

Open questions:

- What is the influence of the drifters' type on the transition matrix ?
- Will it be possible to perform a seasonal (or maybe monthly) evaluation of the transition matrix ?
- How does it compare to high number of artificial drifters or simply density advection using a numerical velocity field or a global circulation model?
- How to "easily" extract the different sets ?

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References II

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