A Lagrangian geography of the deep Gulf of Mexico

Philippe Miron[1], Francisco J. Beron-Vera[1], María J. Olascoaga[1] and Gary Froyland[2]

[1]RSMAS University of Miami, Miami, USA

[2]University of New South Wales, Sydney, Australia

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CARTHE CONSORTIUM for ADVANCED RESEARCH on TRANSPORT of HYDROCARBON in the ENVIRONMENT

CIGoM Consorcio de Investigación del Golfo de México

UNIVERSITY OF MIAMI ROSENSTIEL SCHOOL of MARINE & ATMOSPHERIC SCIENCE

Introduction

Over the last twenty five years, many satellite-tracked surface drifters sampled the surface of the Gulf of Mexico (GoM). From a database of 3300 drifters, we presented a Lagrangian geography of the GoM.

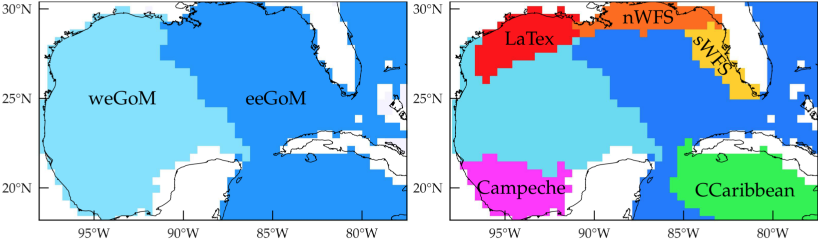

Figure: Published in **miron2017lagrangian**

Introduction

In contrast, very few studies were aimed at the characterization of the deep water global circulation.

RAFOS experiments sponsored by the Bureau of Ocean Energy Management (BOEM) (July 2011 - May 2015) courtesy Alexis Lugo Fernandez:

- 121 floats at 1500 m
- 31 floats at 2500 m
- 4-year mission (floats $\sim$ 2-y mission and are redeployed)

Objectives

Using floats data (trajectories) in the abyssal Gulf of Mexico (GoM):

- subdivide the deep GoM into regions with similar dynamics;
- identify sinks and sources (attractors and basins of attraction);
- predict transport of *passive tracers*.

Theory: flowmap F

Given the drifter paths, we wish to access the map $x \mapsto F(x)$ that determines how the drifters change positions.

If the ocean flow was stationary in some statistical sense, then the velocity could be express has $v(x)$. If this case, we could simply solve the ODE $\dot{x} = v(x)$ to get F, the flow map, and *move around* any function, such has a tracer distribution $f(x)$.

Theory: transfer operator $\mathcal{P}$

This is done by composing f with F^{-1} and it defines a transfer operator $\mathcal{P}$ as the linear operator such that

$$\mathcal{P}f(x) = f \circ F^{-1}(x). \tag{1}$$

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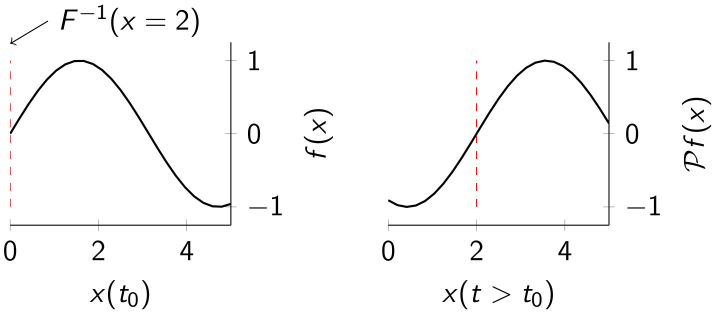
Why F^{-1}? Example, if F moves the distribution 2 units to the right.



Theory: transfer operator $\mathcal{P}$

For F representing a non-autonomous flow as above, $\mathcal{P}f(x)$ is the solution of

$$\partial_t \rho + v \cdot \nabla \rho = 0, \quad \rho(x, t = 0) = f(x). \tag{2}$$

In our case, F is unknown and we only have access to trajectories. We will use a discrete method to approximate $\mathcal{P}$.

Theory: transition matrix

Partition the domain into equal-size bins $\{B_i, \cdots, B_N\}$ (a regular grid) and consider the indicator function of set B_i:

$$\mathbf{1}_{B_i}(x) = \begin{cases} 1 & \text{if } x \in B_i, \\ 0 & \text{if } x \notin B_i. \end{cases} \tag{3}$$

Note that $V_N := \{\mathbf{1}_{B_1}(x), \cdots, \mathbf{1}_{B_N}(x)\}$ is a discrete orthornormal basis w.r.t. the inner product

$$\langle \bullet, \mathbf{1}_{B_i}(x) \rangle = \frac{1}{\mathrm{area}(B_i)} \int_{B_i} \bullet \mathbf{1}_{B_i}(x)\, d^2x = \overline{(\bullet)}^{B_i}. \tag{4}$$

Then, project $f(x)$ on the basis V_N, called Ulam basis (**ulam1960**):

$$\Pi_N f(x) = \sum_{1}^{N} f_i \mathbf{1}_{B_i}(x), \quad f_i = \langle f(x), \mathbf{1}_{B_i}(x) \rangle = \overline{f(x)}^{B_i} \tag{5}$$

where Π_N is the projector.

Theory: how to construct the transition matrix

In a similar manner one can project $\mathcal{P}$ on V_N:

$$(\Pi_N \mathcal{P})_{ij} = \frac{1}{\mathrm{area}(B_j)} \int_{B_j} \mathcal{P}\mathbf{1}_{B_i}(x) \cdot \mathbf{1}_{B_j}(x)\, d^2x$$
$$= \frac{\mathrm{area}(B_i \cap F^{-1}(B_j))}{\mathrm{area}(B_i)} =: P_{ij}. \qquad (6)$$

The entries of P can be viewed as transitional probabilities of moving from B_i to B_j (Markov Chain with bins $\equiv$ states):

$$P_{ij} = \frac{\#\text{ of particles in } B_i \text{ that are mapped to } B_j}{\#\text{ of particles in } B_i}. \qquad (7)$$

Theory: transition matrix

P_{ij} gives us the action of F at a coarse-grained level given by the partition. This introduces diffusion proportional to the size of B_i and is solution of:

$$\partial_t \rho + v \cdot \nabla \rho = D \nabla^2 \rho \quad \text{with } \rho(x, t = 0) = f(x), \tag{8}$$

with $D \propto \mathrm{area}(B_i)$.

Application of the transition matrix

One can push forward discrete representations of $f(x)$:

$$\mathbf{f} = (f_1, \cdots, f_N), \tag{9}$$

by right-multiplication by P:

$$\begin{aligned} f^{(1)} &= f\,P \\ f^{(2)} &= f^{(1)}\,P = f\,P^2 \\ f^{(k)} &= f\,P^k \end{aligned} \tag{10}$$

Similarly, we push backward an initial distribution using $P^\top$.

Eigenvectors analysis

It is also of interest to identify when a distribution $\mathbf{f}$ is almost invariant:

$$\mathbf{f} \approx \mathbf{f}\, P \tag{11}$$

This is available from the *eigenspectrum* inspection of P (Froyland et al., 2012).

If in the matrix P:

- all states *communicate*;
- no state occurs *periodically*.

P has a limiting distribution $\mathbf{p} = \mathbf{p}P$.

Note: $\mathbf{p}$ is a left eigenvector of P with eigenvalue $\lambda = 1$ ($\mathbf{p}\lambda = \mathbf{p}P$). Because of row-stochasticity of P, the corresponding right eigenvector is $\mathbb{1}$, i.e., $P\mathbb{1} = \mathbb{1}$.

Attractors and basin of attractions

Any distribution $f_{\mathbb{1}}$ supported on the right eigenvector $\mathbb{1}$ will converge to $\mathbf{p}$ as the number of applications of P tends to infinity, i.e., $\lim_{n\to\infty} f_{\mathbb{1}} P^n = \mathbf{p}$.

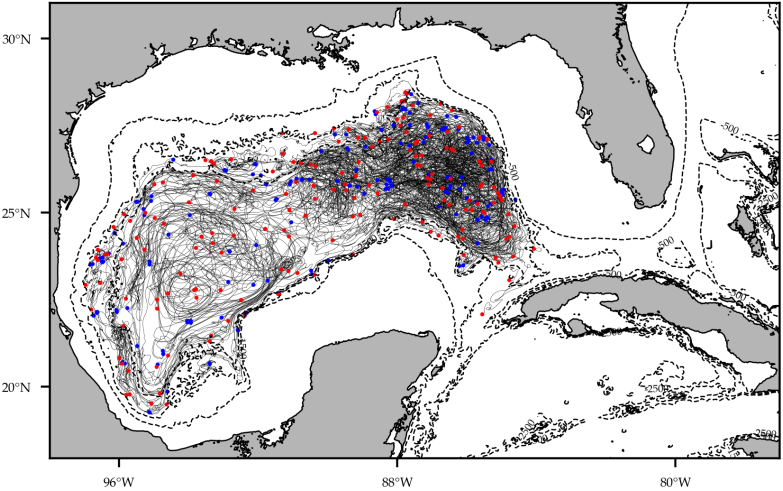
For $\lambda = 1$:

- right eigenvector of P is the basin of attraction
- left eigenvector of P is the attractor

This motivates the idea that regions where trajectories converge and their basins of attraction are encoded in the eigenvectors of the transition matrix P with eigenvalues ($\lambda \approx 1$) (**froyland2014well**).

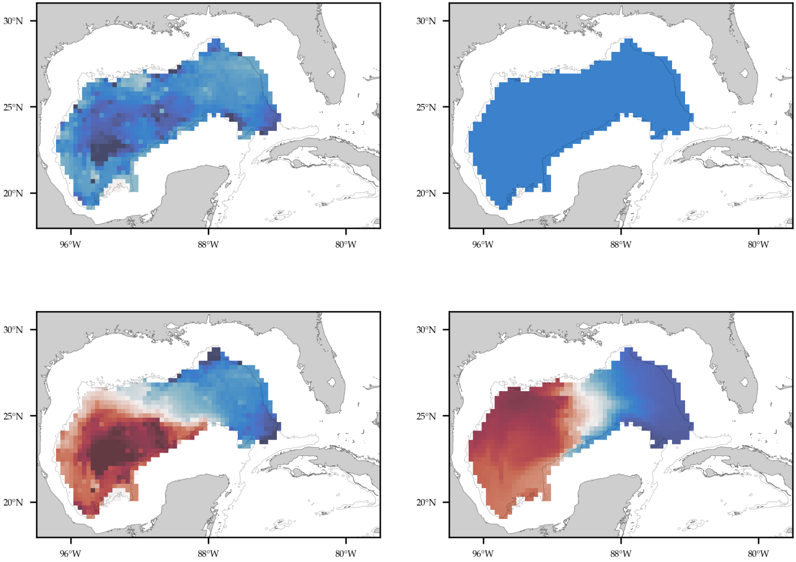
Data

Trajectories of the RAFOS cover the area under 1500 m of the Gulf of Mexico.



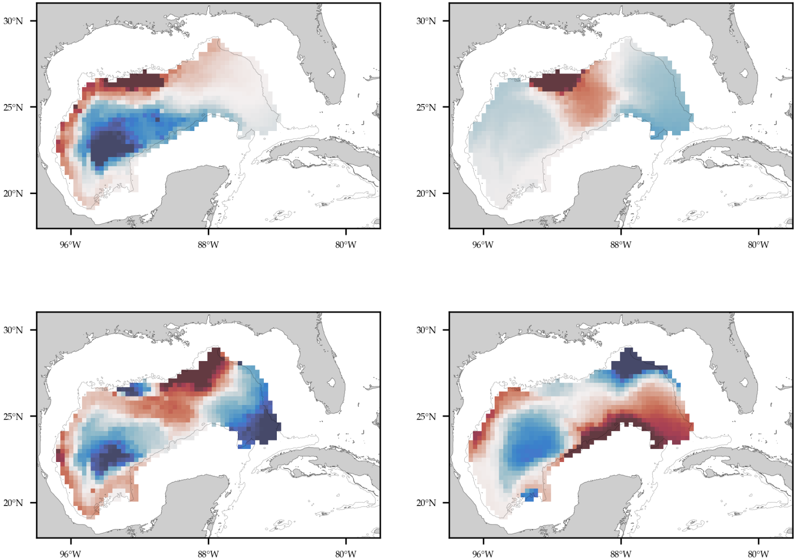
Eigenvectors

Eigenvectors associated with $\lambda_1 = 1$ and $\lambda_2 = 0.9953$.



Eigenvectors

Eigenvectors associated with $\lambda_3 = 0.9832$ and $\lambda_5 = 0.9712$.



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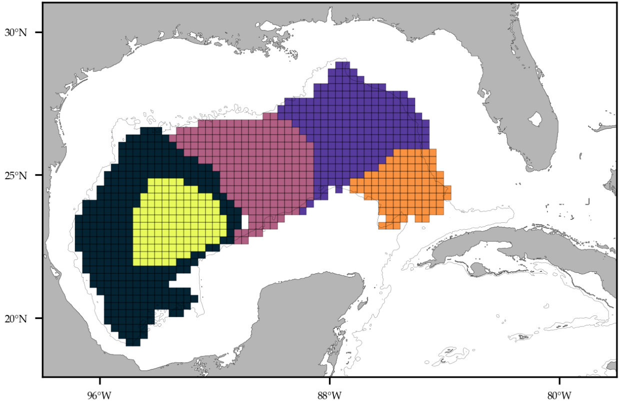
Combination of the basins of attraction from the top right eigenvectors (by thresholding).

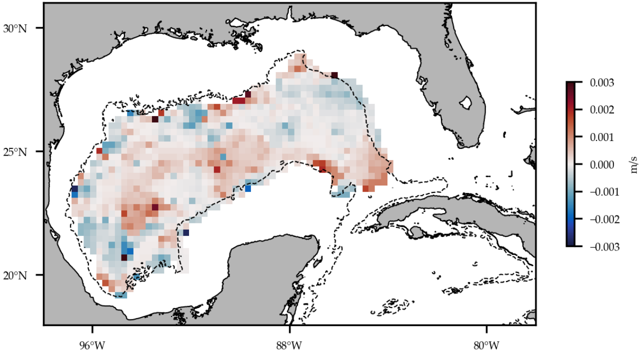


Residence timescale: 7-y left (black) and right (purple), 1-y middle (pink), 0.5-y gyre (yellow) and bottom right (orange)

Upwelling and Downwelling

From incompressibility, accumulation can be used to approximate vertical velocity w:

$$h_{t_1} = h_{t_0} \frac{\mathrm{area}_{t_0}}{\mathrm{area}_{t_1}} \quad w \sim \frac{h_{t_1} - h_{t_0}}{dt}. \tag{12}$$



The vertical component is about $\sim 1\%$ of the maximum horizontal velocity ($u_{max} = 0.2739$m/s, $v_{max} = 0.1790$m/s).

Comparison with experimental data (**ledwell2016dispersion**)

Tracer evolution from the infamous Deepwater Horizon oil spill.

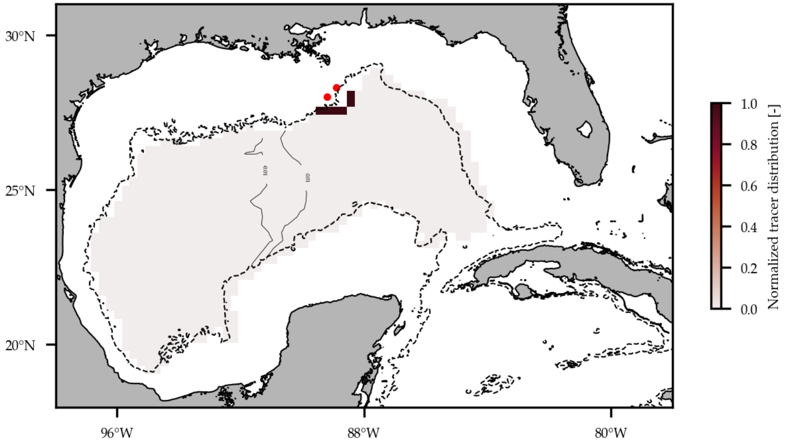


Figure: Initial location of the tracers

Comparison with experimental data (**ledwell2016dispersion**)

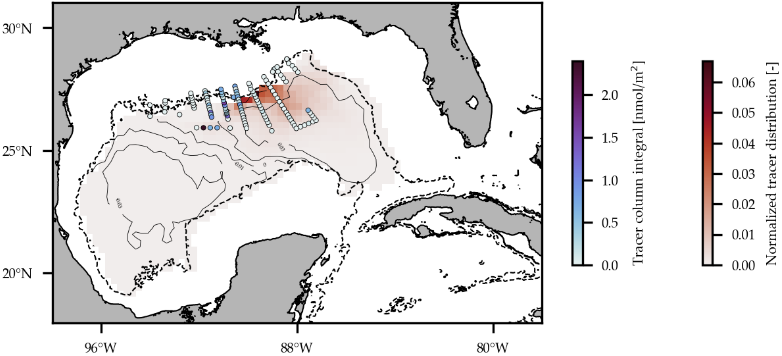


Figure: Evolution after 4 months

Comparison with experimental data (**ledwell2016dispersion**)

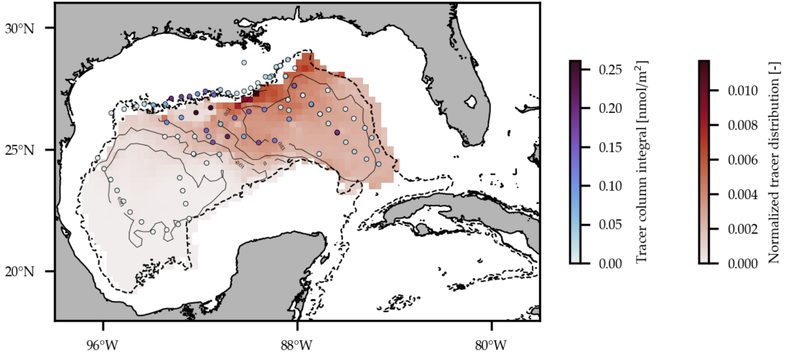


Figure: Evolution after 12 months

Thank you!

Future plans:

- plastic sources and convergence zones
- take account inertial effects (see next talk!)

Any questions?

References I

Froyland, G. et al. (2012). "Three-dimensional characterization and tracking of an Agulhas Ring". In: *Ocean Modelling* 52-53, pp. 69–75, 2012.