A Lagrangian geography of the deep Gulf of Mexico

Philippe Miron[1], Francisco J. Beron-Vera[1], María J. Olascoaga[1], Gary Froyland[2], J. Sheinbaum[3] and P. Pérez-Brunius[3]

[1]RSMAS University of Miami, Miami, USA

[2]University of New South Wales, Sydney, Australia

[3]CICESE, Ensenada, Mexico

Ocean Sciences Meeting on Wednesday, February 14 2018

CIGoM
Consorcio de Investigación del Golfo de México

UNIVERSITY OF MIAMI
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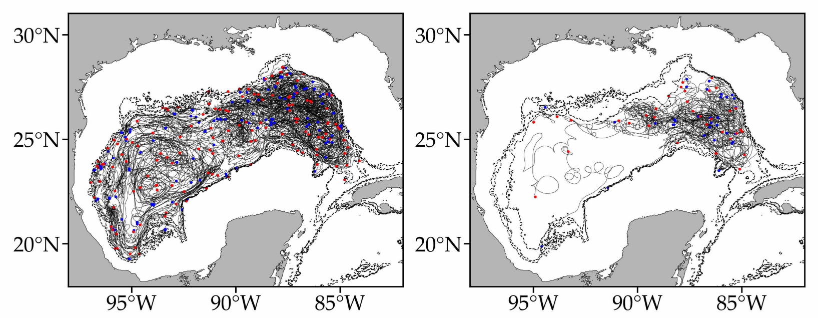
Introduction

RAFOS experiment sponsored by the Bureau of Ocean Energy Management (July 2011 - May 2015)[1]:

- 4-year-long program (floats $\sim$ 2-y mission)
- 121 floats at 1500 m
- 6 profiling floats with RAFOS technology at 1500 m
- 31 floats at 2500 m



[1] Publicly available data sets compiled by WOCE Subsurface Float Data Assembly Center (WFDAC).

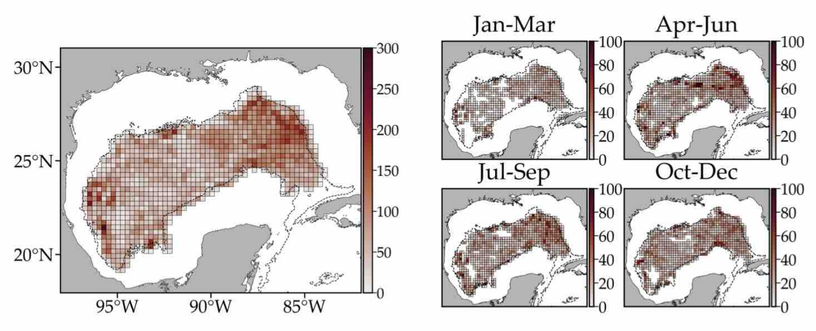
Objectives

Using floats data (trajectories) in the abyssal Gulf of Mexico (GoM):

- subdivide the deep GoM into regions with similar dynamics;
- identify almost invariant regions and their respective timescale;
- assess connectivity.

Seasonality of the RAFOS Data

The data coverage isn't sufficient for full seasonal analysis but assuming time homogeneity it is enough to build a Markov-Chain model (Maximenko, Hafner, and Niiler, 2012; Miron et al., 2017; McAdam and Sebille, 2018).



Theory: how to construct the transition matrix

By partitioning the domain X into a grid of N regular connected boxes $\{B_1, \ldots, B_N\}$ and with large number of initial conditions we can estimate the entries:

$$\mathcal{P} \approx P_{ij} = \frac{\#x \text{ in } B_i \text{ at any time } t \text{ and in } B_j \text{ at } t+T}{\#x \text{ in } B_i \text{ at any time } t}, \qquad (1)$$

which are transitional probabilities of moving from B_i to B_j. It defines a **Markov Chain** (with bins $\equiv$ states) of the dynamics.

Timescale T is fix at 7-d which is larger then the decorrelation scale of 5-d and enough to allow interbins connection.

Application of the transition matrix

One can push forward discrete representations of $f(x)$:

$$\mathbf{f} = (f_1, \cdots, f_N), \tag{2}$$

under left-multiplication by P:

$$\begin{aligned} f^{(1)} &= f\, P \\ f^{(2)} &= f^{(1)}\, P = f\, P^2 \\ f^{(k)} &= f\, P^k \end{aligned} \tag{3}$$

Eigenvectors analysis

It is also of interest to identify when a distribution $\mathbf{f}$ is almost invariant:

$$\mathbf{f} \approx \mathbf{f}\, P \tag{4}$$

This is available from the *eigenspectrum* inspection of P (Froyland, Horenkamp, et al., 2012).

If in the matrix P:

- all states *communicate*;
- no state occurs *periodically*.

P has one $\lambda = 1$ a limiting distribution $\mathbf{p} = \mathbf{p}P$.

Note: $\mathbf{p}$ is a left eigenvector of P (row-stochastic matrix) with eigenvalue $\lambda = 1$.

$$\mathbf{p}\lambda = \mathbf{p}P$$
$$\mathbb{1}\lambda = P\mathbb{1}$$

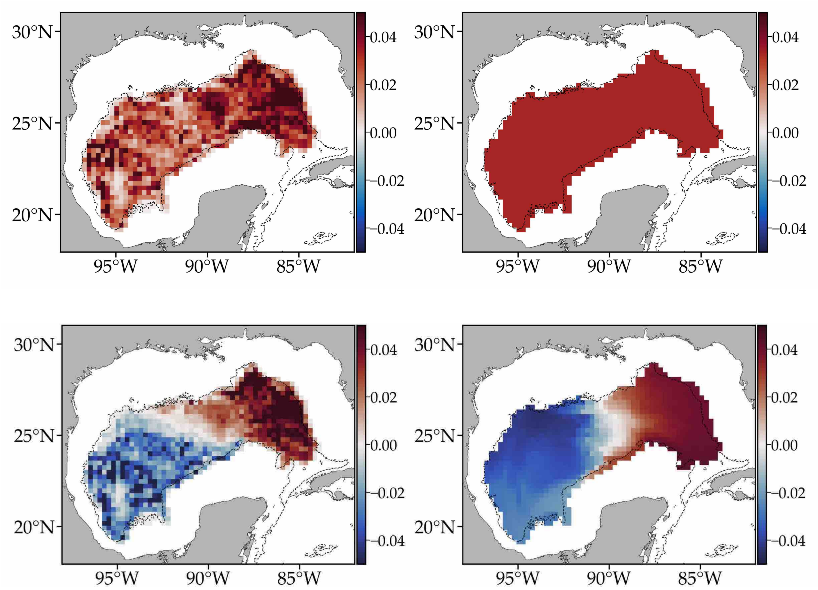
Attractors and basin off attractions

Motivates the idea that regions where trajectories converge and their basins of attraction are encoded in the eigenvectors of the transition matrix P with eigenvalues ($\lambda \approx 1$) (Froyland, Stuart, and van Sebille, 2014).

- **right eigenvector** of P is the basin of attraction (**constrains connectivity!**)
- **left eigenvector** of P is the attractor or almost-invariant region

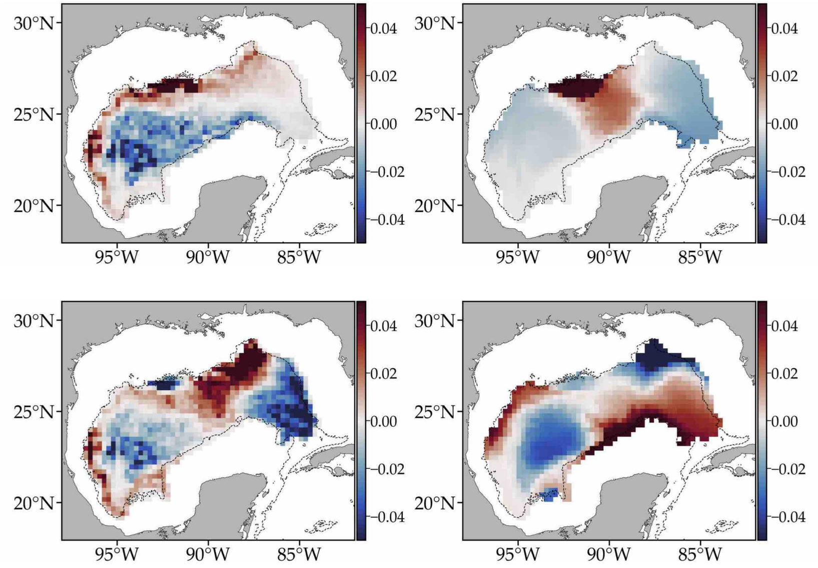
Eigenvectors

Eigenvectors associated with $\lambda_1 = 1$ and $\lambda_2 = 0.9953$.



Eigenvectors

Eigenvectors associated with $\lambda_3 = 0.9832$ and $\lambda_5 = 0.9712$.



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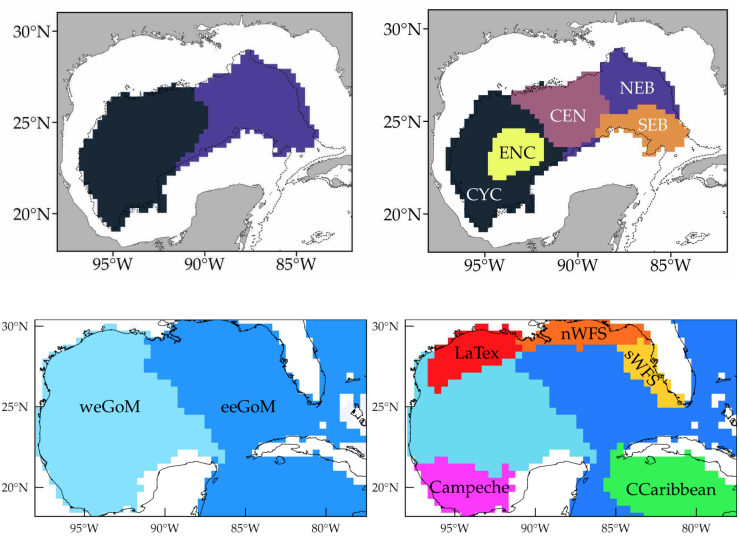
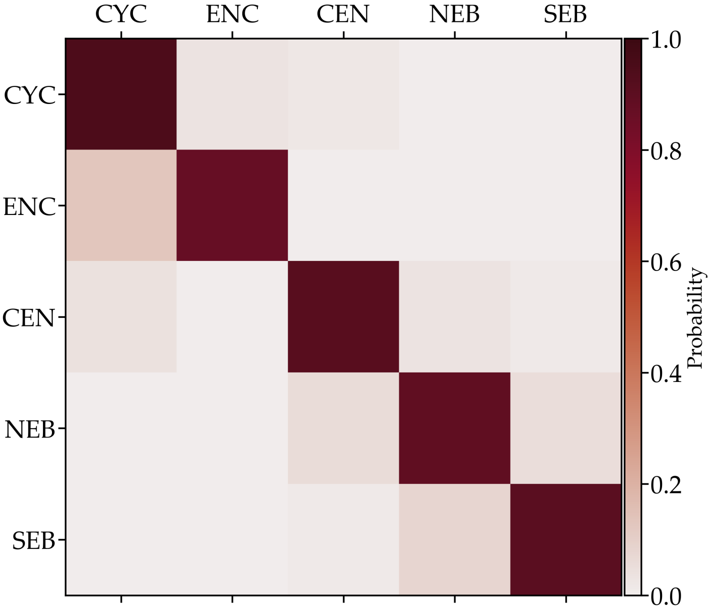
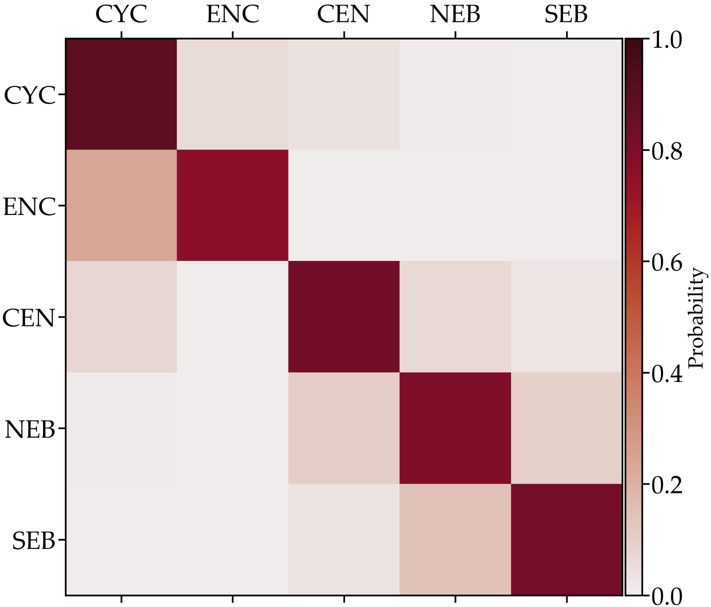
Combination of the basins of attraction from the top right eigenvectors (by thresholding).



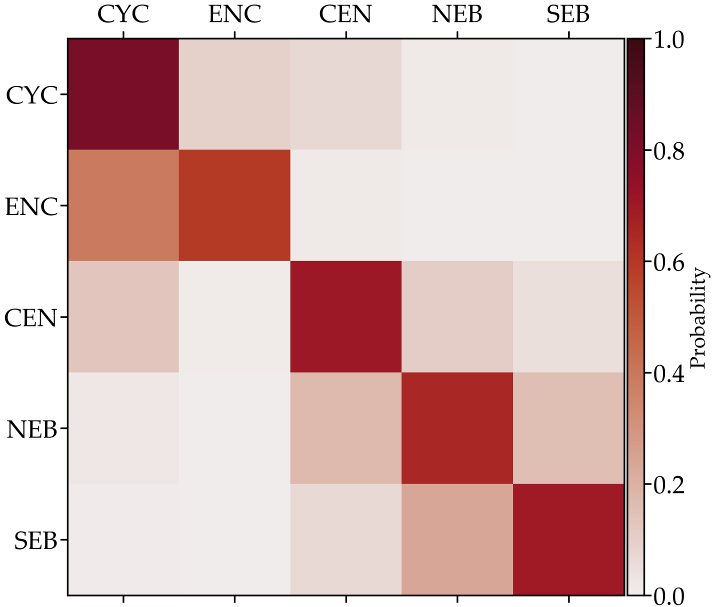
Connectivity matrix (1 week)



Connectivity matrix (2 weeks)



Connectivity matrix (4 weeks)

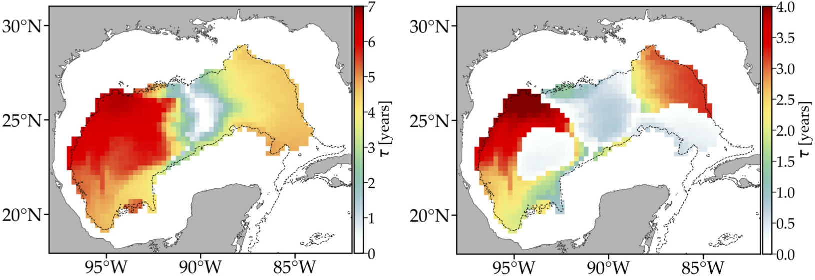


Residence time

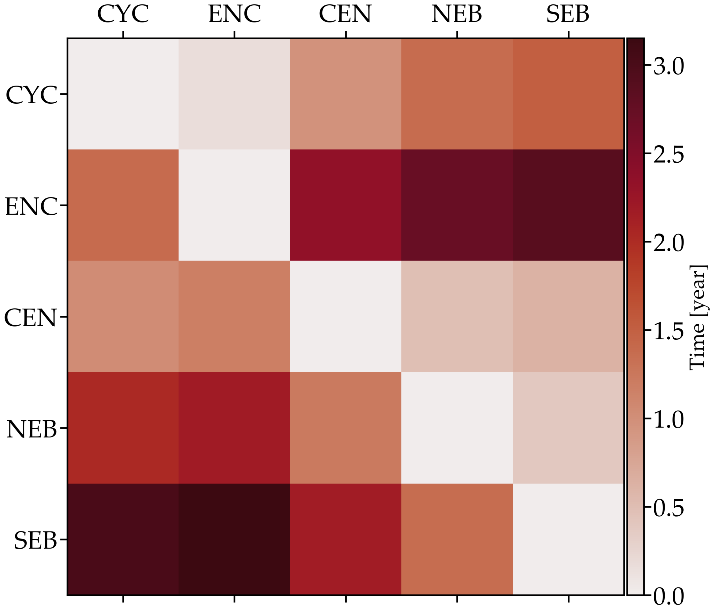
The time τ for a trajectory in box B_i to move out of A, also known also as the mean time to hit the complement of A (Norris, 1998).

$$(\mathsf{Id} - P|_A)\tau / T = \mathbf{1}, \tag{5}$$

The time on average to reach a given province starting from any province can be computed using (5) with A set to the target province. We can see the cyclonic motion on the western region (Pérez-Brunius et al., 2018).

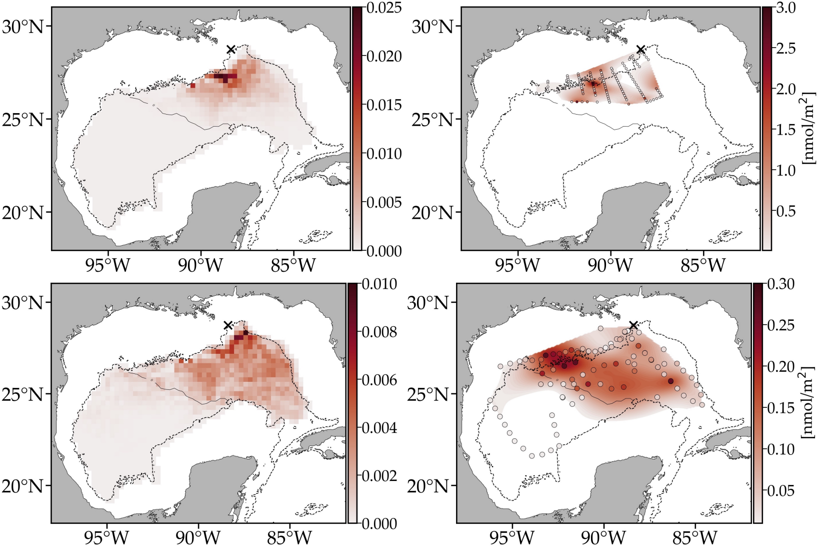


Mean expected hitting time (complement of A in (5))



Validation with experimental data (Ledwell et al., 2016)

Tracer mostly spreads along the continental slope and across Eastern bassin.



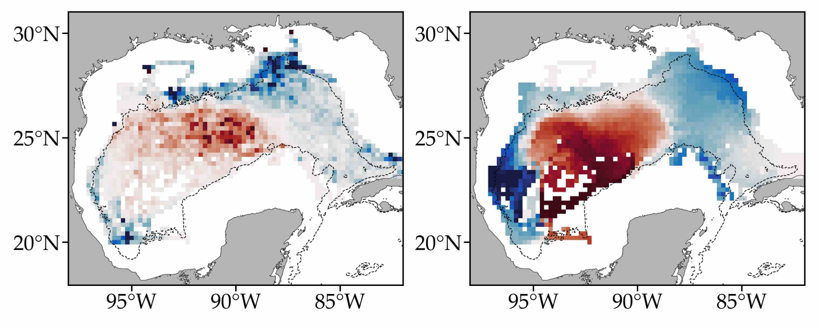
Conclusion

- Assuming 3-d volume conservation, vertical flows over 1 yr is $\overline{w} = 0.2242\ \text{m/d}$
- Flow is mostly horizontal and ventilated from the Caribbean Sea (also explains why no float escape?)
- Fast spreading (≈ 1 yr over the eastern part) as observed by Ledwell et al., 2016
- Main partition is also reveal by the Argo floats

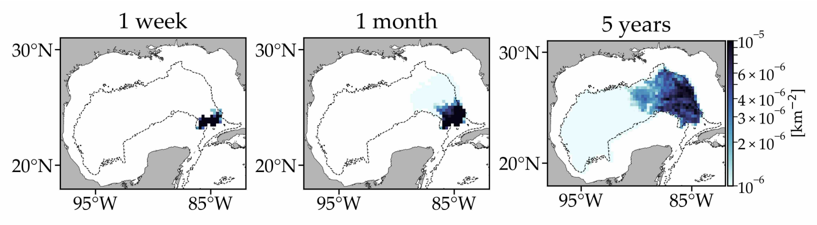
Future plans:

- Evaluation of global circulation from surface drifters (GDP) and deep water floats (RAFOS, SOFAR & ARGO)

Main partition from Argo floats

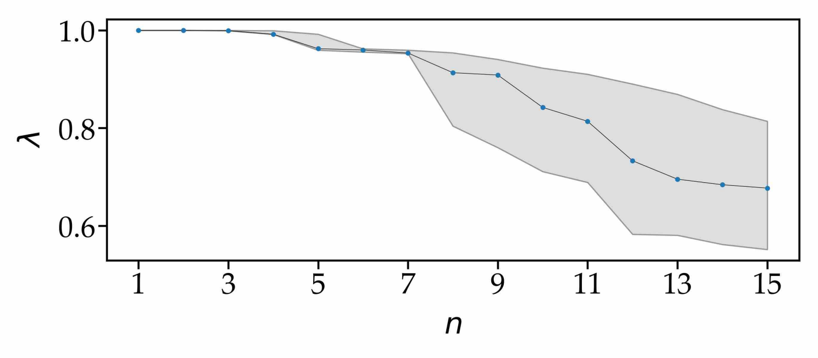


Push forward in the Eastern corner of the domain



Eigenvalues cut-off

Look at the effect of random noise in the float trajectories on the eigenvalues.



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