Lagrangian geographies of the Gulf of Mexico

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LAPCOD, Venezia, 2019

CIGoM
Consorcio de Investigación del Golfo de México

GULF OF MEXICO RESEARCH INITIATIVE

Objectives

Using trajectories of floats and drifters in the Gulf of Mexico (GoM):

- Subdivide the GoM and the deep GoM into regions with similar dynamics
- Identify almost invariant regions and their respective timescale
- Assess connectivity
- Forecast density evolutions (oil spill, plastic, larvae, etc)

Data sets for surface and deep GoM

Drifters

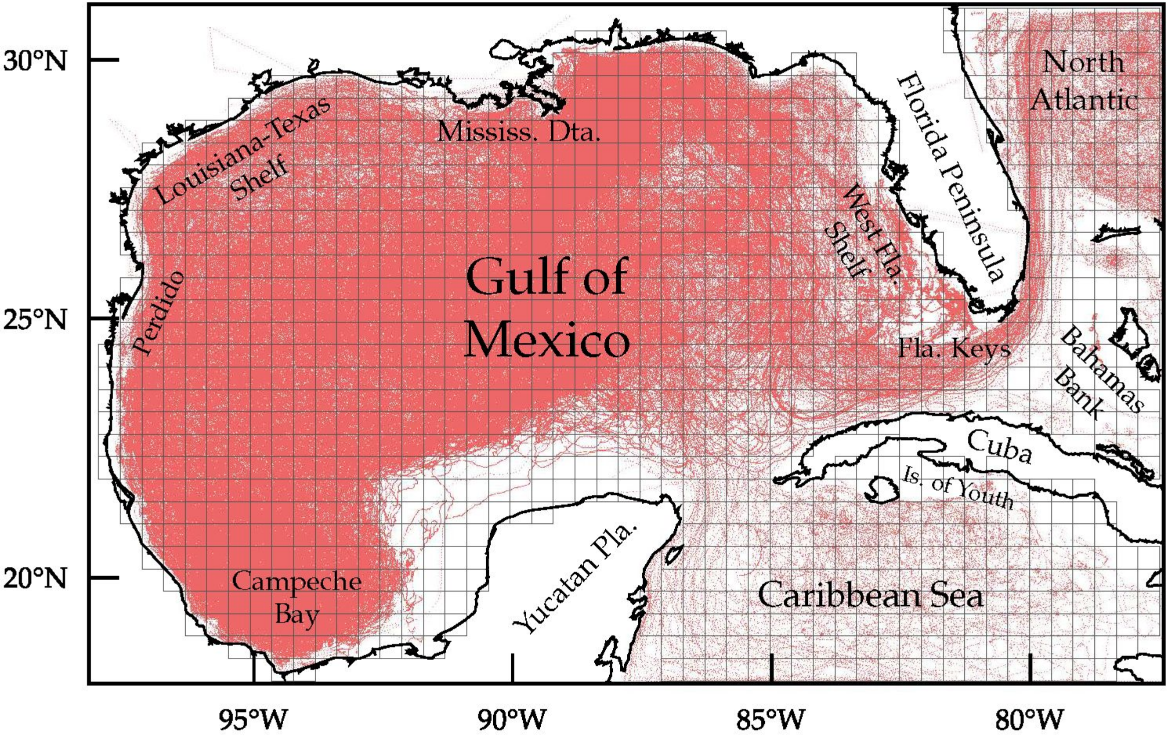
- 3500+ drifter trajectories from several different sources 1992–2018 (GDP[1], CARTHE[2], CICESE, SCULP, AOML[2], USCG[2])

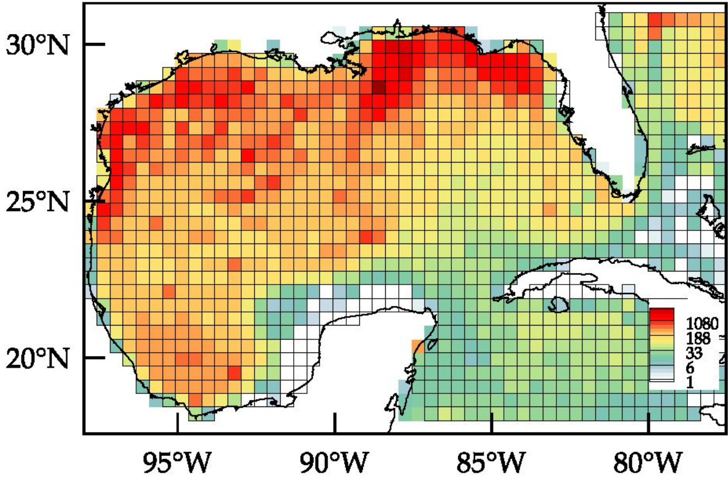
RAFOS experiment sponsored by BOEM (July 2011–May 2015)[3]:

- 121 floats at 1500 m
- 6 profiling floats with RAFOS technology at 1500m
- 31 floats at 2500 m

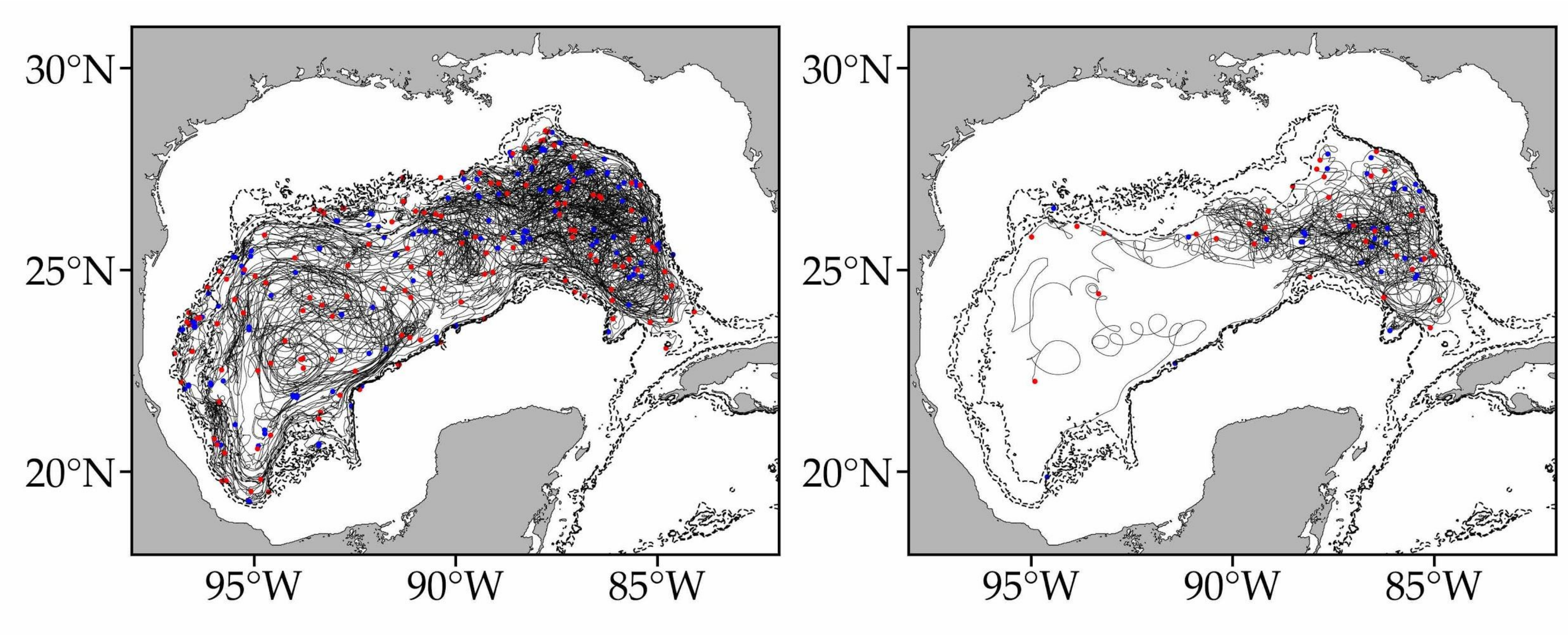
Data publicly available
[1] Physical Oceanography Division Global drifters (PhOD GDP)
[2] Gulf of Mexico Research Initiative Information & Data Cooperative (GRIIDC)
[3] WOCE Subsurface Float Data Assembly Center (WFDAC)

Drifter trajectories and density



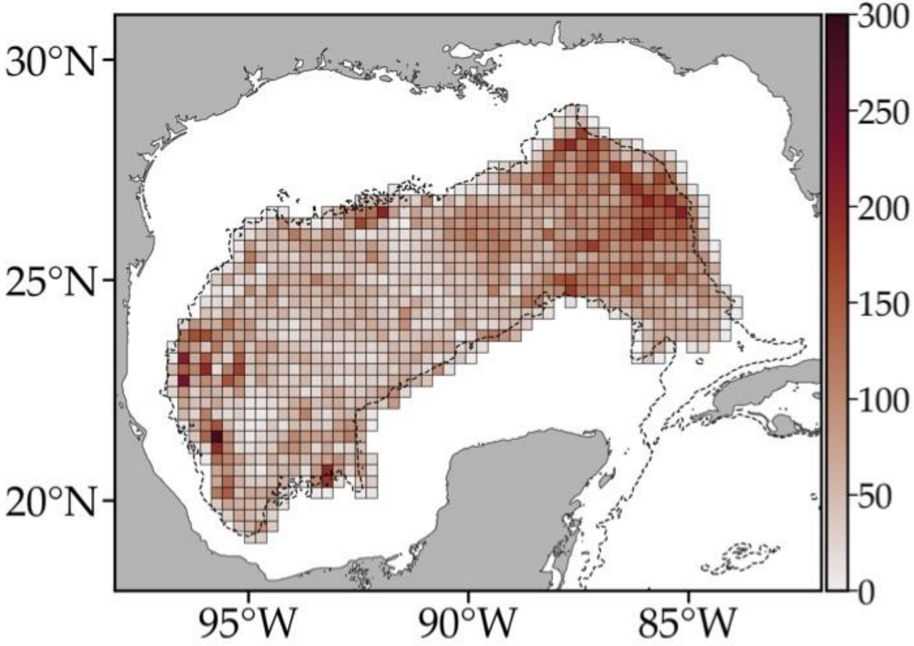


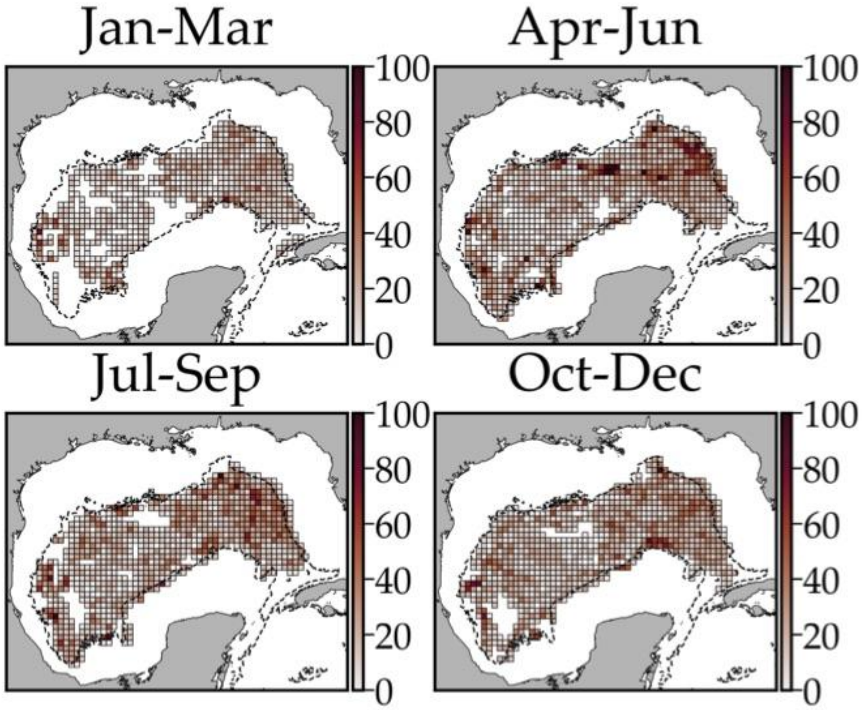
RAFOS float trajectories



1500 m (left) and 2500 m (right)

RAFOS float density





Complete data set (left) and seasonal data sets (right)

Transfer Operator

- Focus on the evolution of probability densities rather than individual trajectories
- transition probabilities are described by a stochastic kernel *K(x,y)*
- *f(x)* is a probability density
  - Larger or equal to 0
  - Integral on the domain is 1
- Evolution of the density forward in time

$$\mathcal{P} f(y) = \int_X K(x, y) f(x) \, \mathrm{d}\, x$$

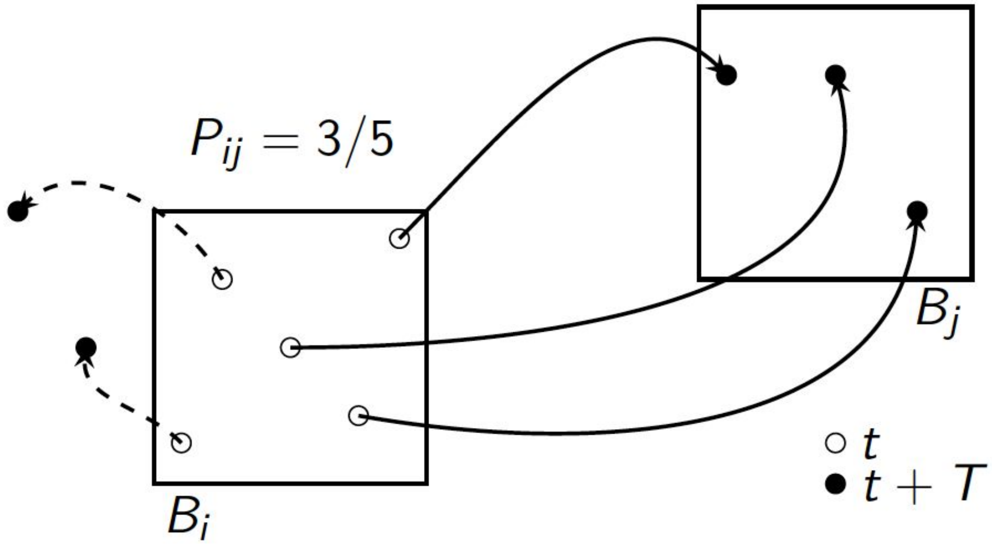
Discrete version of the Transfer Operator

- Cover the domain with N connected boxes (B_1, B_2, …, B_N)
- Projected functions in $L^1(X)$ onto a finite-dimension space V_N
- Perron-Frobenius theorem
- Probability of moving from bin i to j:

$$P_{ij} = \mathrm{prob}[\xi_{t+T} \in B_j \mid \xi_t \in B_i] = \frac{\int_{B_j} \int_{B_i} K(x, y)\, \mathrm{d}\, x\, \mathrm{d}\, y}{\mathrm{area}(B_i)}$$

Transition matrix

$$P_{ij} \approx \frac{\#\text{ points in } B_i \text{ at } t \text{ that evolve to } B_j \text{ at } t+T}{\#\text{ points in } B_i \text{ at } t}$$



Evolve a density (pushforward)

- Left multiplication of the discrete density *f(x)* with the transition matrix
- Each time evolve the density T days
- If P is row-stochastic, the mass is conserved and $\sum f_i = 1$.

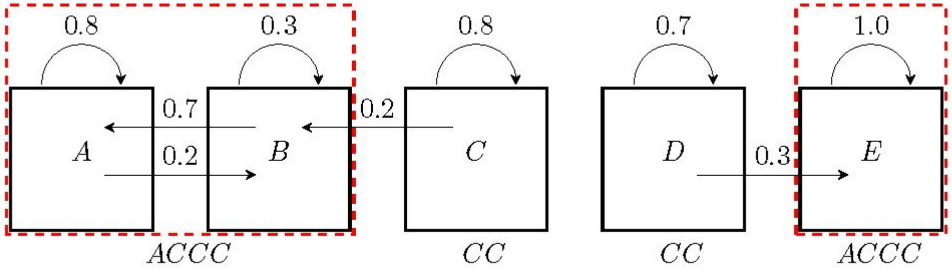
$$\mathbf{f}^{(k)} = \mathbf{f}P^k, k = 1, 2, \ldots$$

Eigenspectrum

Regions where trajectories converge and their basins of attraction are encoded in the eigenvectors of the transition matrix (Froyland, Stuart, and van Sebille 2014).

- Right eigenvector of $\boldsymbol{P}$ is the basin of attraction (constraints connectivity)
- Left eigenvector of $\boldsymbol{P}$ is the attractor or almost-invariant region

Example with a simple 5 states problem



$\mathcal{P} =$

	A	B	C	D	E
A	0.8	0.2	0	0	0
B	0.7	0.3	0	0	0
C	0	0.2	0.8	0	0
D	0	0	0	0.7	0.3
E	0	0	0	0	1.0

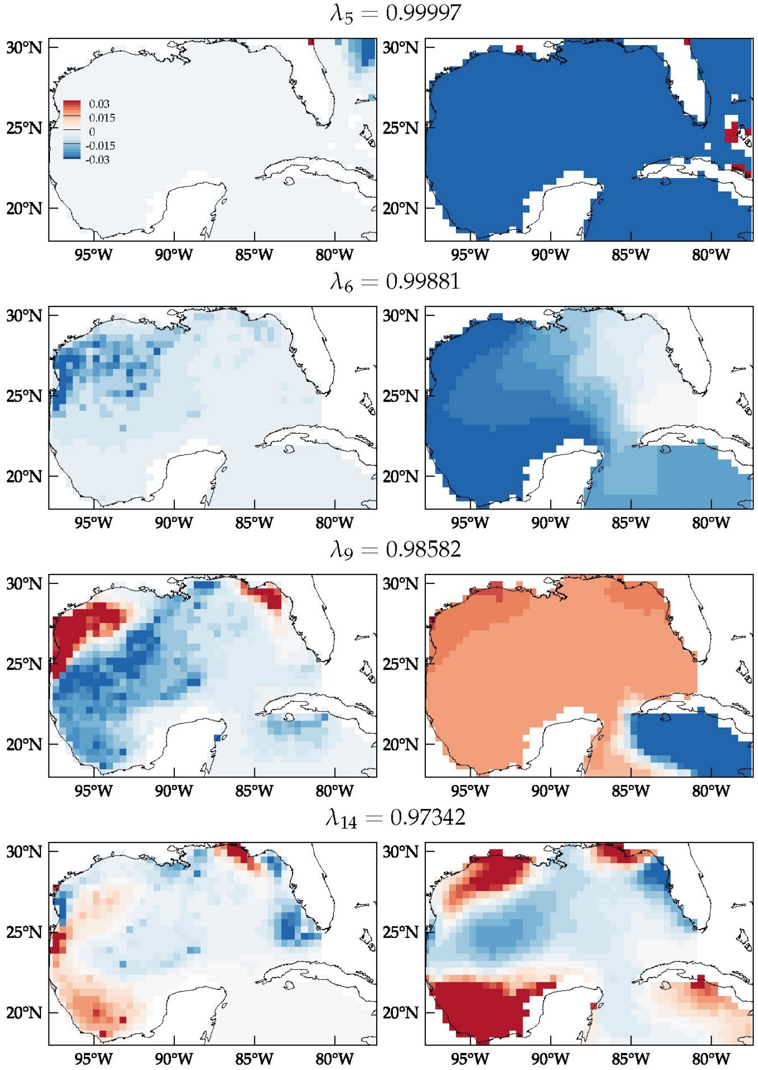
Example with a simple 5 states problem

$$L_1^T = \begin{pmatrix} 0.83 \\ 0.55 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad L_2^T = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad R_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad R_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

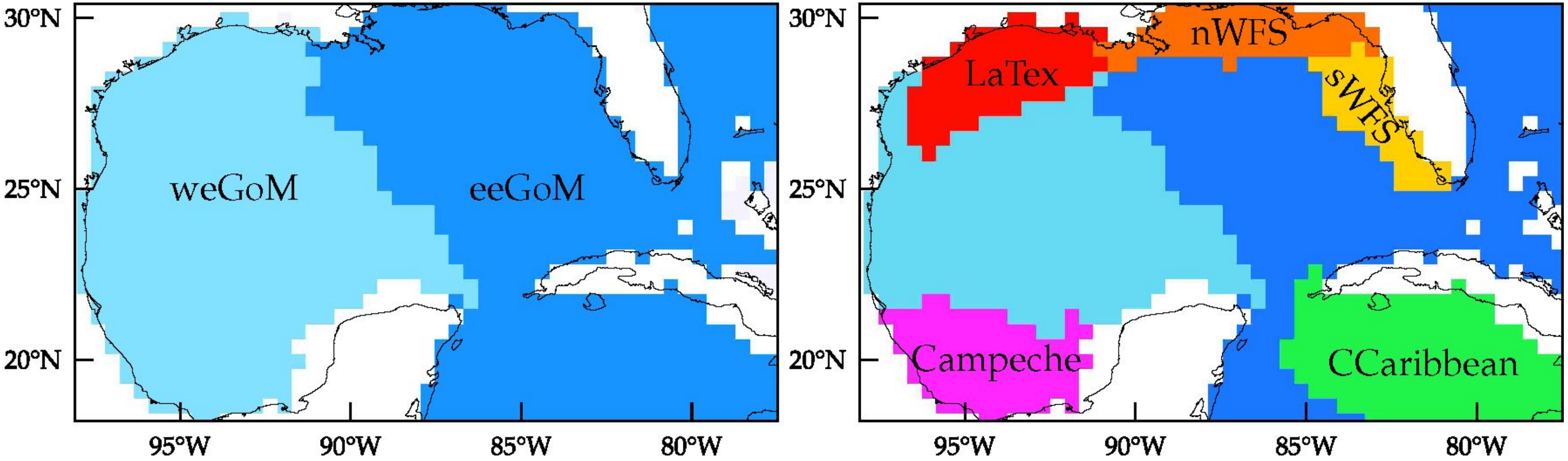
- A, B are the attractors
- E is another attractor

- A, B, C basins of attraction
- D, E basins of attraction

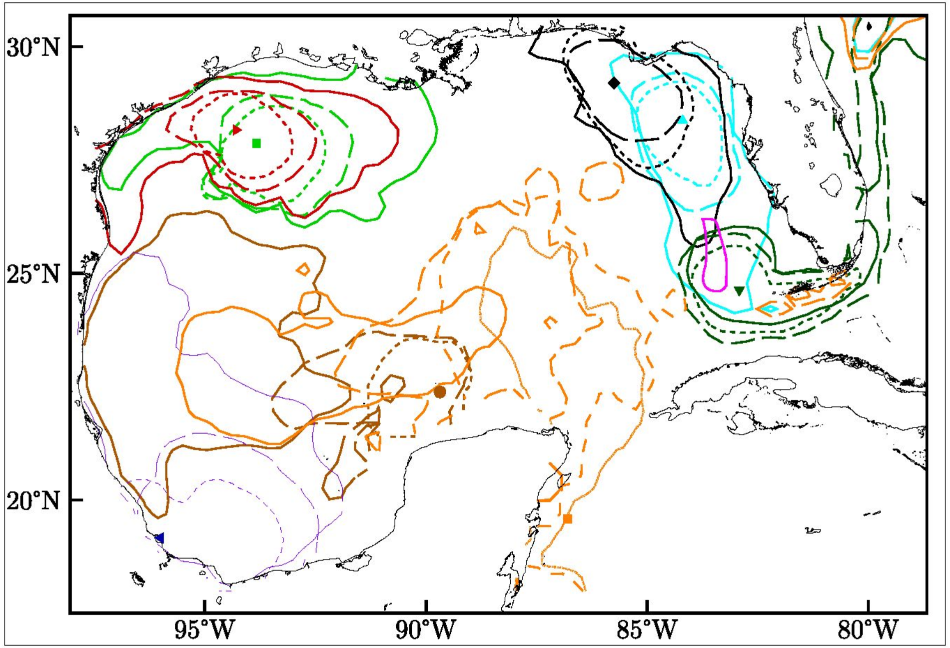
Eigenvectors

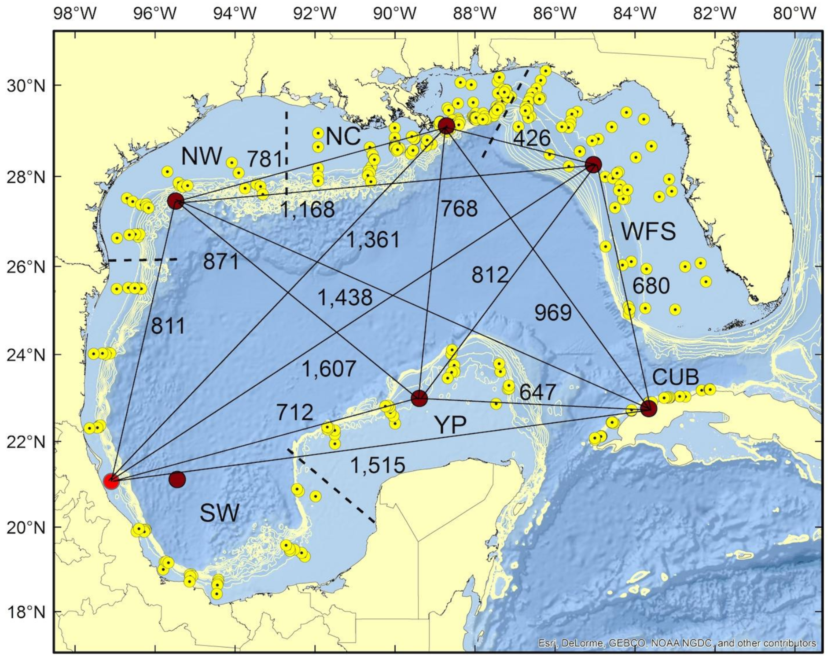


Lagrangian geography



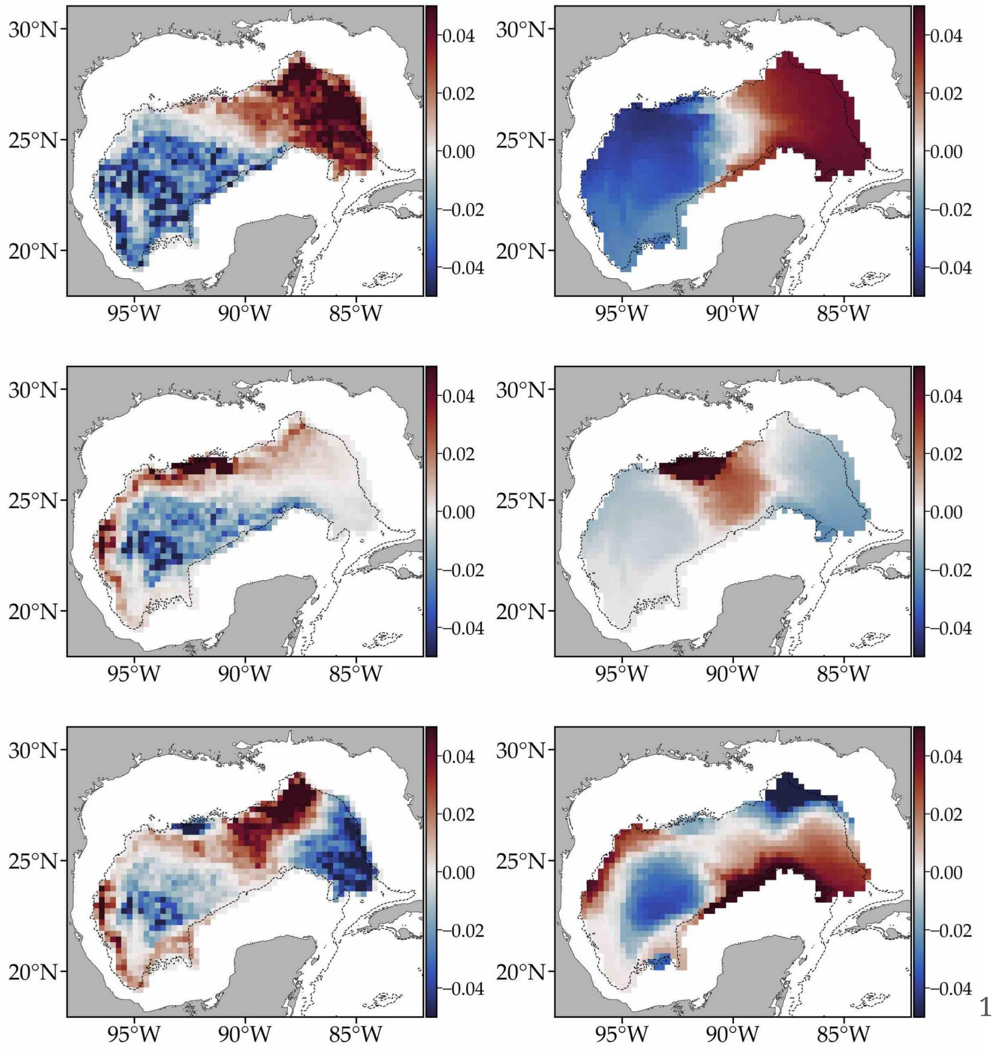
Applications (Olascoafa, 2018 and Murawski, USF)



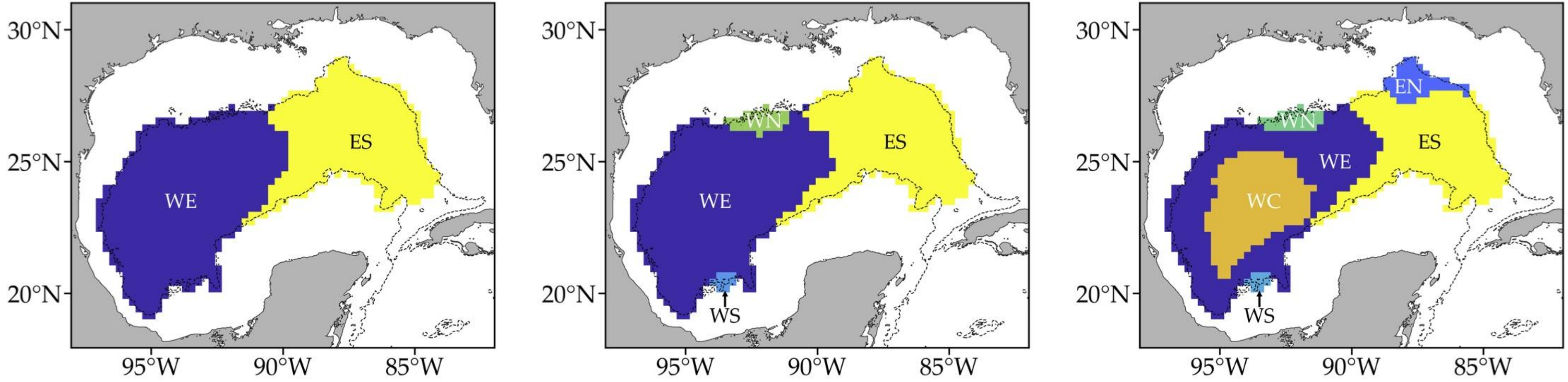


Connectivity of different Reefs in the GoM (left) and Group of tagged adult fish (right)

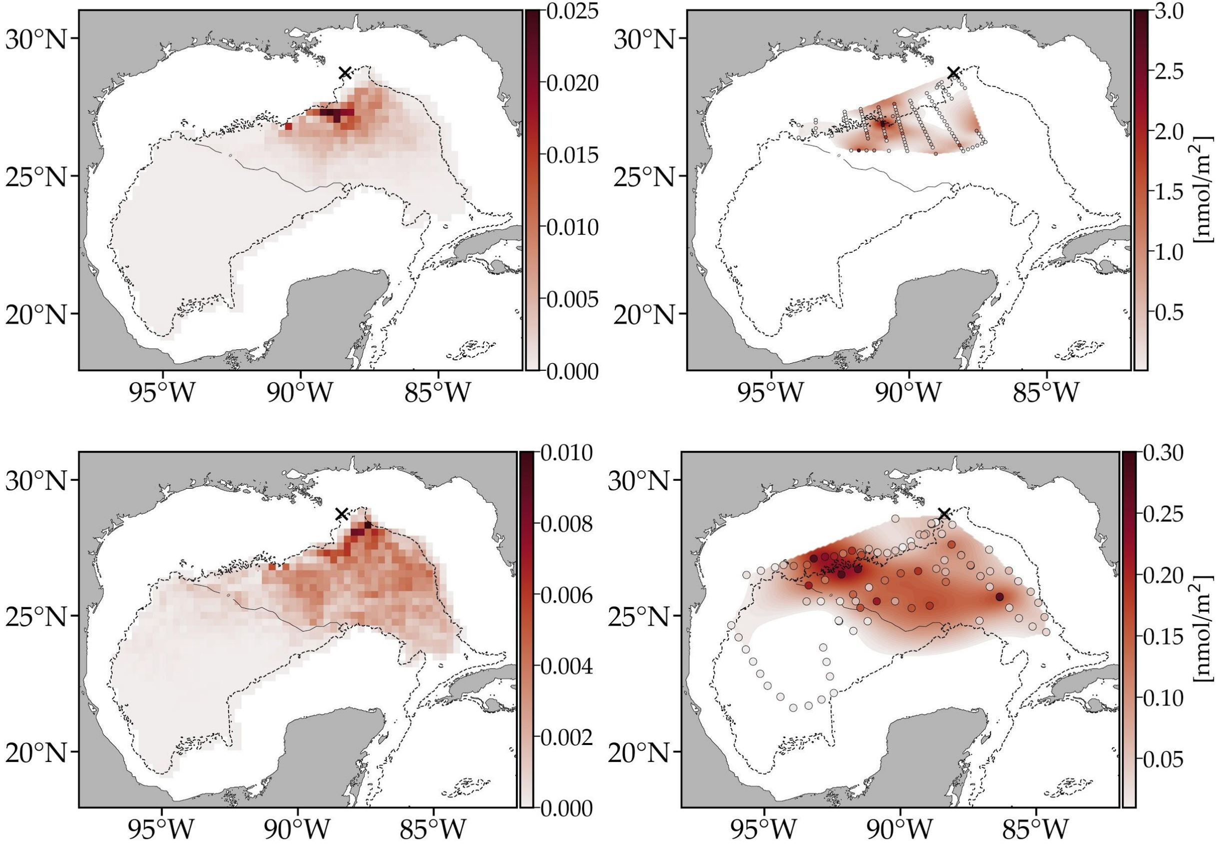
Eigenvectors



Lagrangian geography



Comparison with tracer dispersion (Ledwell 2012)



Markov-chain-inspired search for MH370

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LAPCOD, Venezia, 2019

Catastrophe

The disappearance in the Indian ocean of Malaysian Airlines flight MH370 is one of the biggest aviation mysteries.

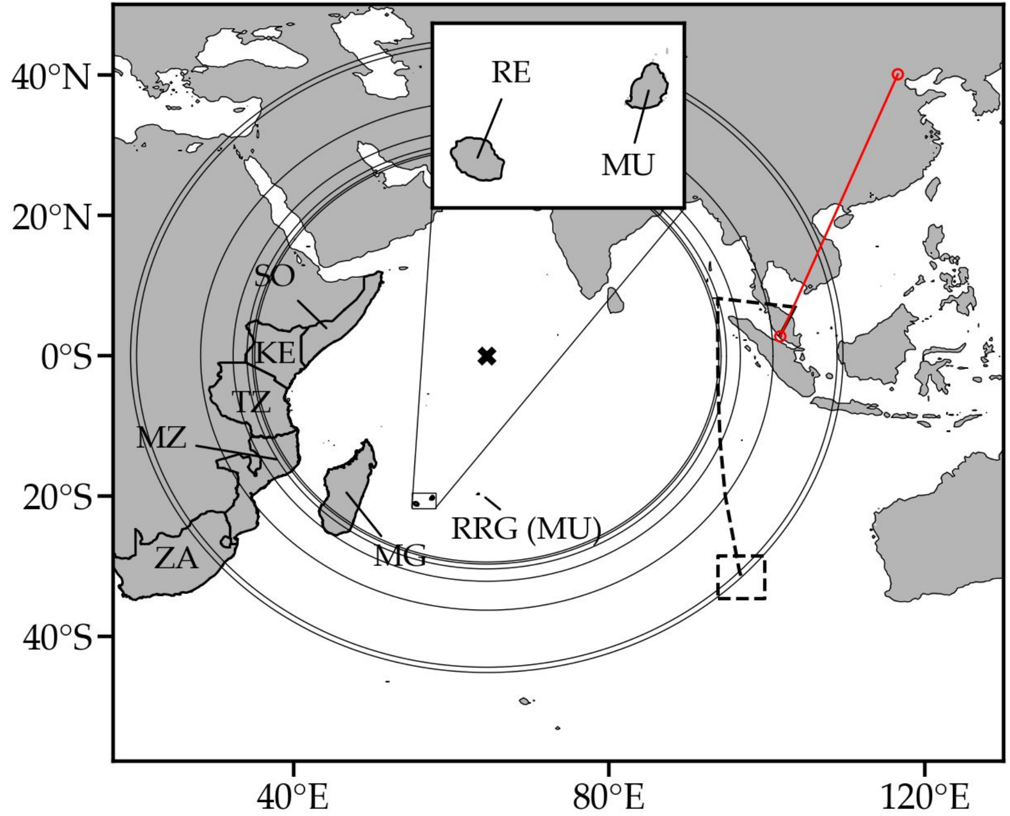
- second deadliest incident involving a Boeing 777 (227 passengers and 12 crew members)
- most expensive search in aviation history ($155 million)
  - 2 years and 4 months
  - 4 vessels equipped with deep-water vehicles, sonars and cameras
  - underwater scan of a 120 000 km2 search area

Time of March 8, 2014 (UTC time)

- 16:42: takeoff from KUL to PEK with an expected arrival 22:30 (00:42–06:30 MYT)
- 17:06: aircraft final transmission (position and remaining fuel)
- 17:19: last verbal communication from the pilot to air traffic control
- 17:22–18:22: aircraft is observed by civilian and military radar
- until the transponder stopped working (or turned off)
- 19:41–00:19: seven log-on acknowledgements between the
- plane and the satellite
- 01:15: aircraft did not respond to a status request

23:24 (07:24 MYT): Malaysia Airlines stated that contact with the plane was lost at 17:21 and that search operations are launched.

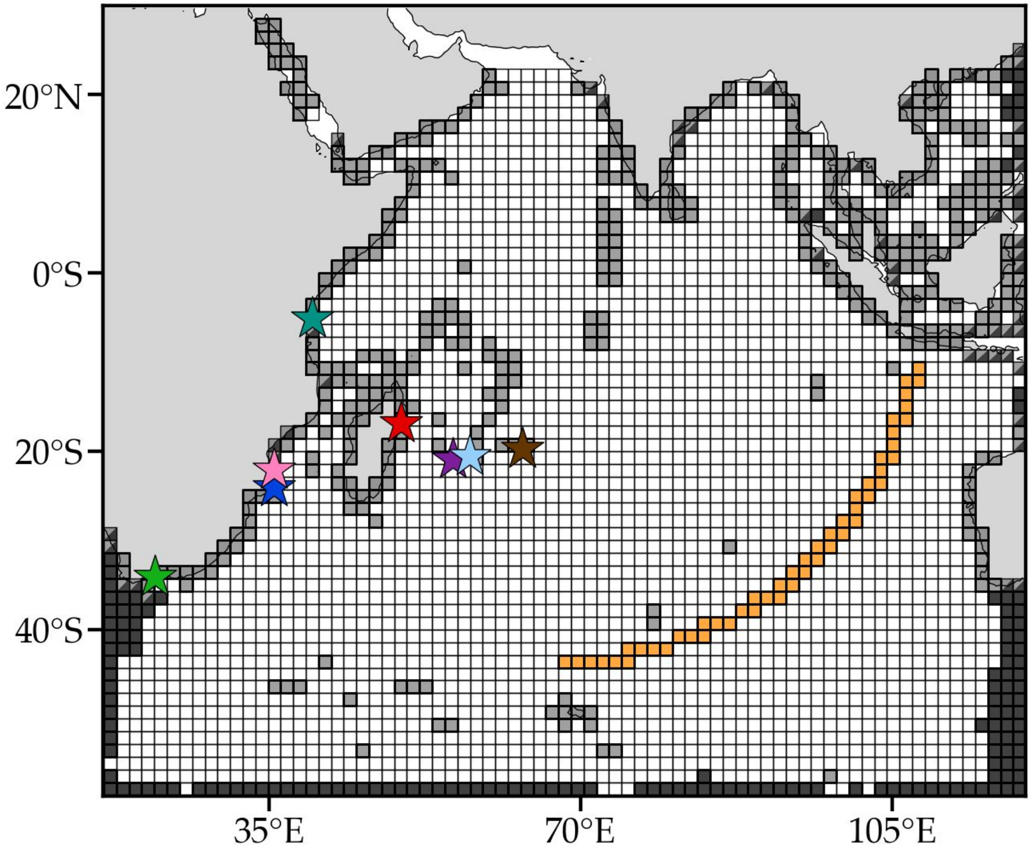
Path reconstruction from satellite *ping* signal



First debris reached Reunion Island

508 days after the crash of the airplane

Confirmed debris beaching (508–838 days)



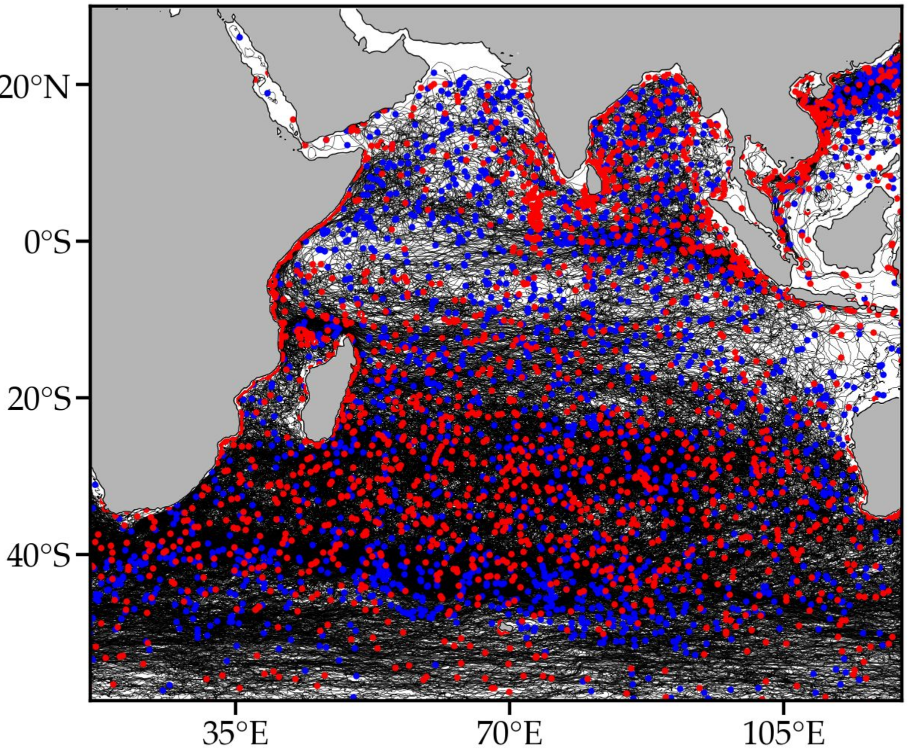
Goal

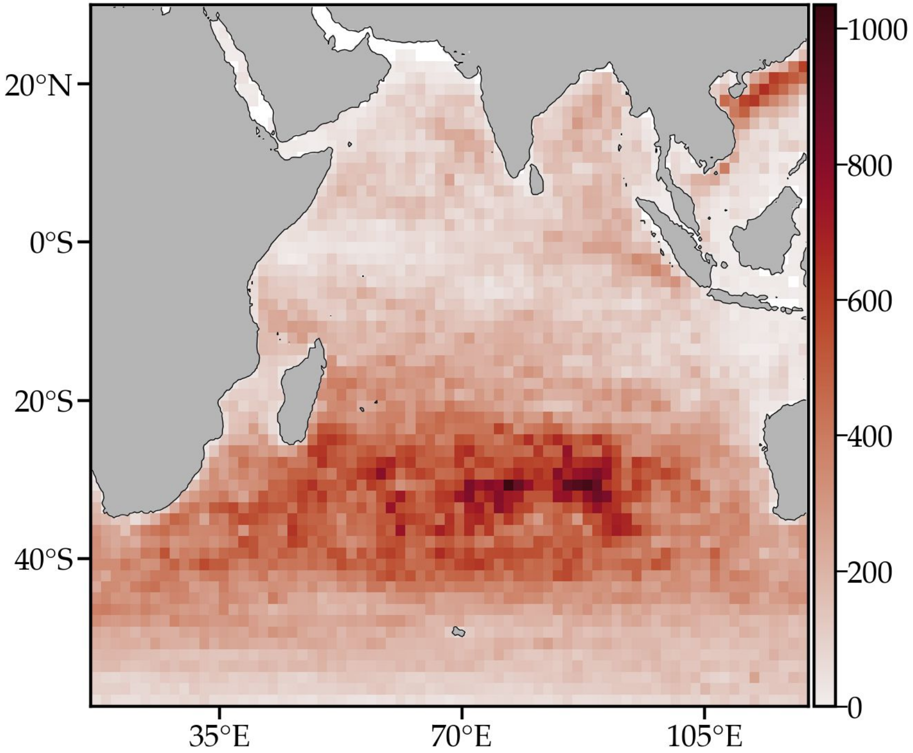
Using undrogued drifters, which are subject to similar dynamics as floating debris, build a model to estimate the airplane crash site.

Model will utilize all available information:

- Last known satellite arc
- Debris beaching time and location

Undrogued drifter in the Indian Ocean





Trajectoires since 1979 (left) and density of 226 data points per bin (right)

Modelisation and adaptation

- Transition time: T = 5 days
- Resolution : 2.5° x 2.5°
- Seasonality of Monsoon : three transition matrices (summer, winter, spring/fall)
- Mass exiting the domain (to a nirvana states)
- Beaching probability equals to the land ratio

Long-term behavior

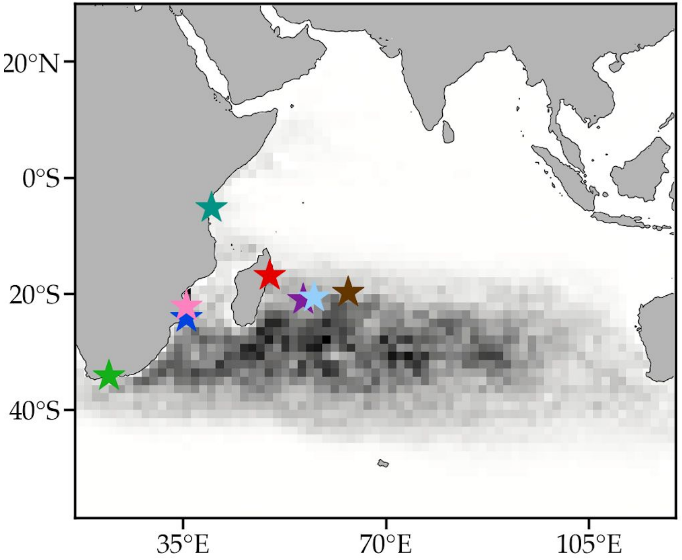
Long-term behavior can be observed from the spectral analysis of the autonomous season-aware matrix:

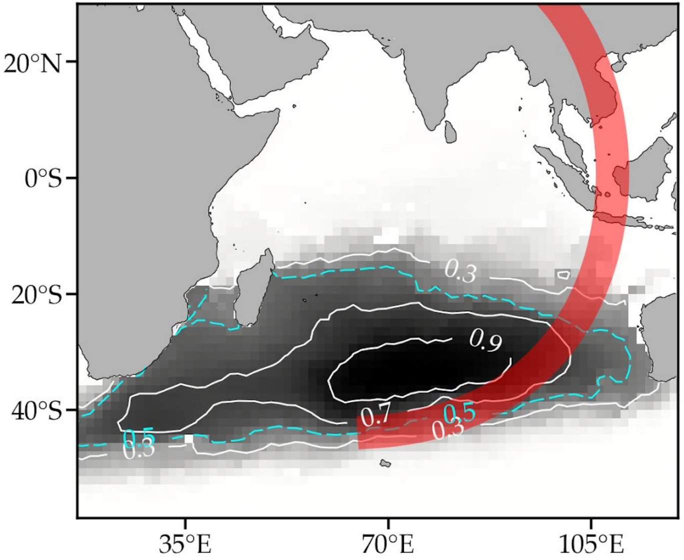
$$P = P_W^{18} \cdot P_{SF}^{18} \cdot P_S^{18} \cdot P_{SF}^{18}$$

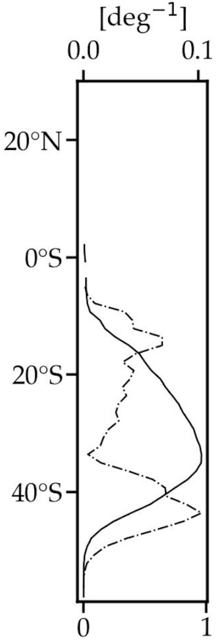
Note that : 18·T = 90 days ~ 3 months

Quasistationary distribution

First approximation of the search area is obtain from the eigenvectors of the combined seasonal P along the satellite arc between the cyan dashed contour (~3600 km).







Bayesian analysis

Estimates the posteriori probability distribution of the crash site that matches the available observations (debris location, time, satellite arc).

Probability to beach at the time of a debris

The time $\boldsymbol{\tau}$ and the probabilities *p* taken until hitting target set *b* for the first time from the crash site $\boldsymbol{c}$ (any state on the arc).

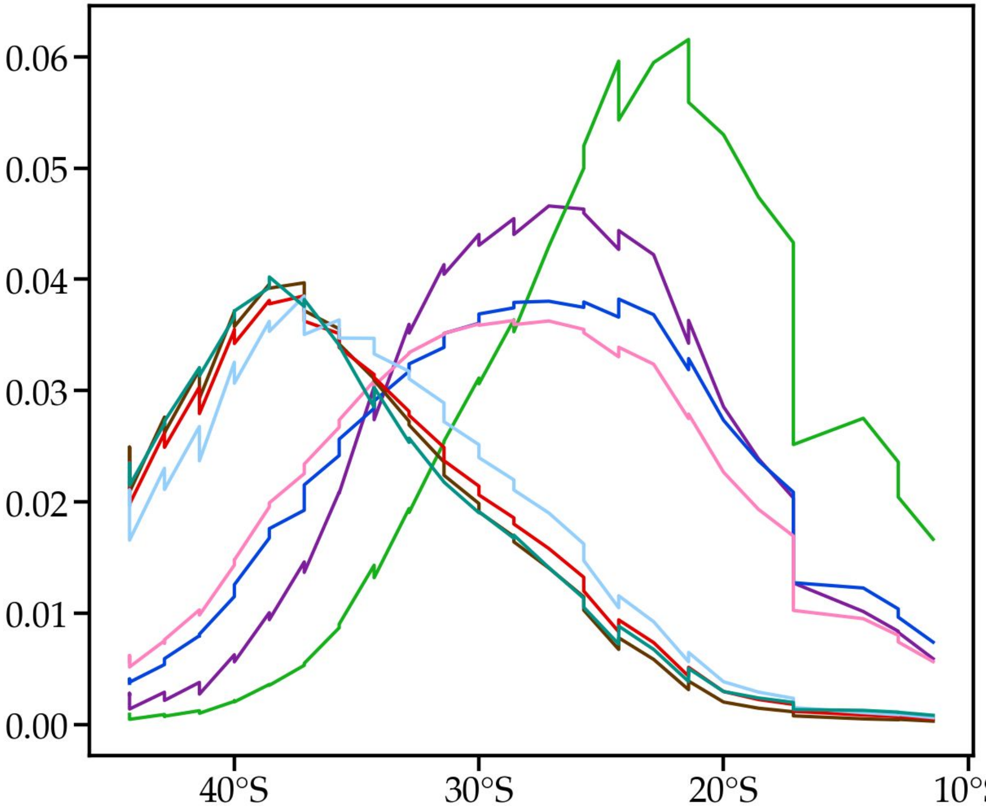
$$\tau^b = \inf_{k \geq 0} \{t + kT : \epsilon_{t+kT} = b\}$$

$$p_c^b(k) = \mathbf{1}_c P^k \cdot \mathbf{1}_b$$

$$\mathrm{prob}[\tau^b = t{+}kT] = \begin{cases} p_c^b(k) = 0 & \text{if } k = 0 \\ p_c^b(k) - p_c^b(k-1) & \text{if } k > 0 \end{cases}$$

Probability for each debris along the satellite arc

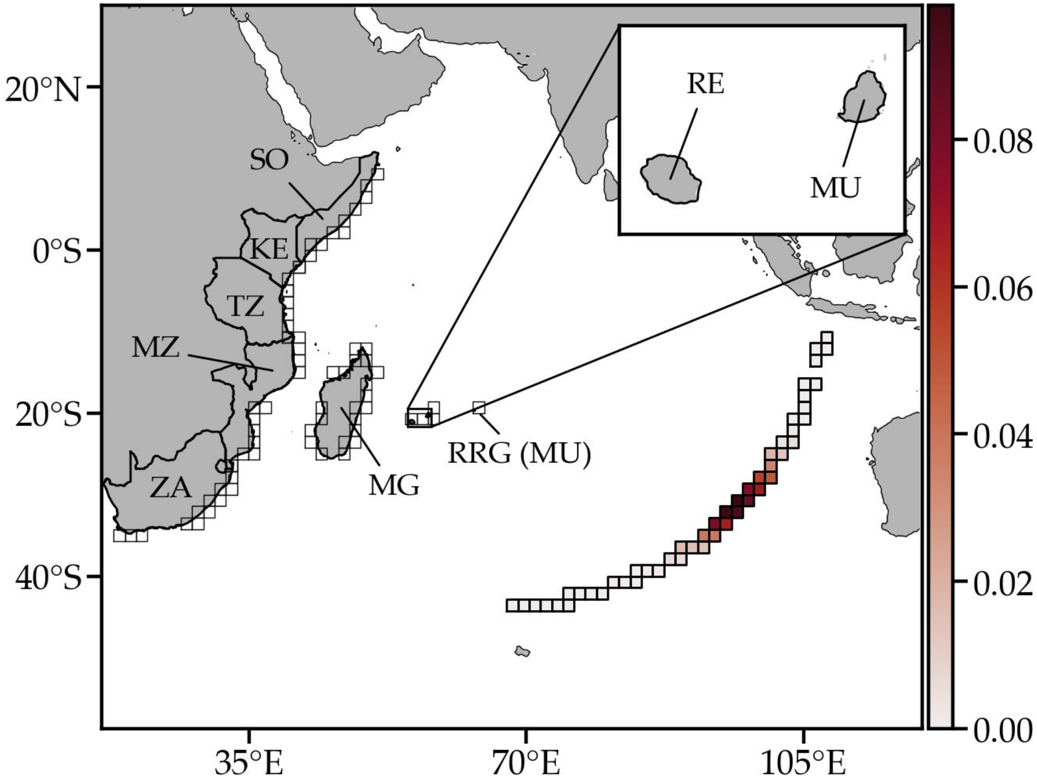
- Two groups (37°N & 25°N)



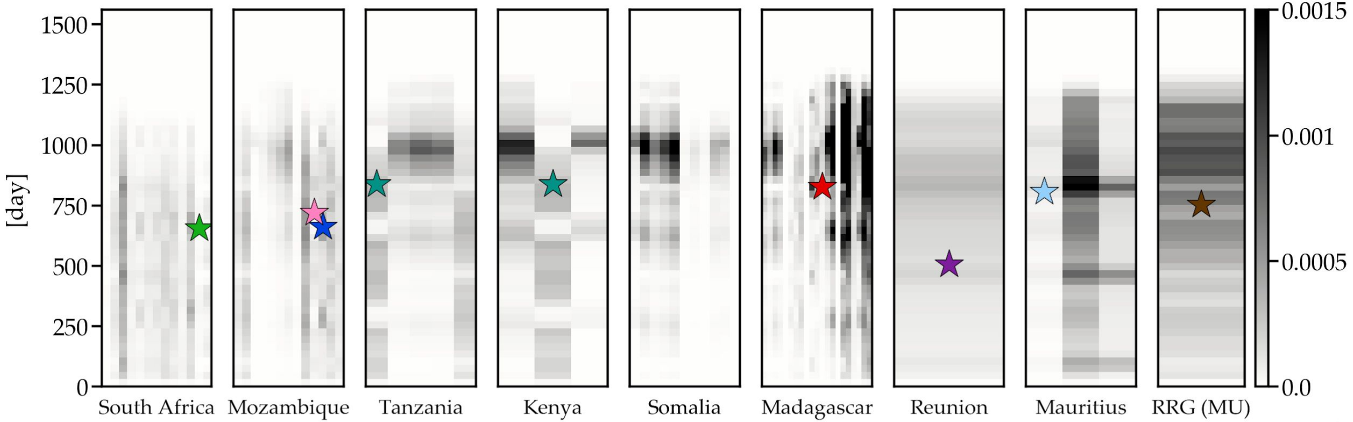
Bayesian analysis

The posterior distribution is the product of all debris probability distributions.

$$p(t^{\mathbf{b}}|c) = \prod_{b \in \mathfrak{B}} p(t^b|c)$$



Heatmap of the beaching probabilities

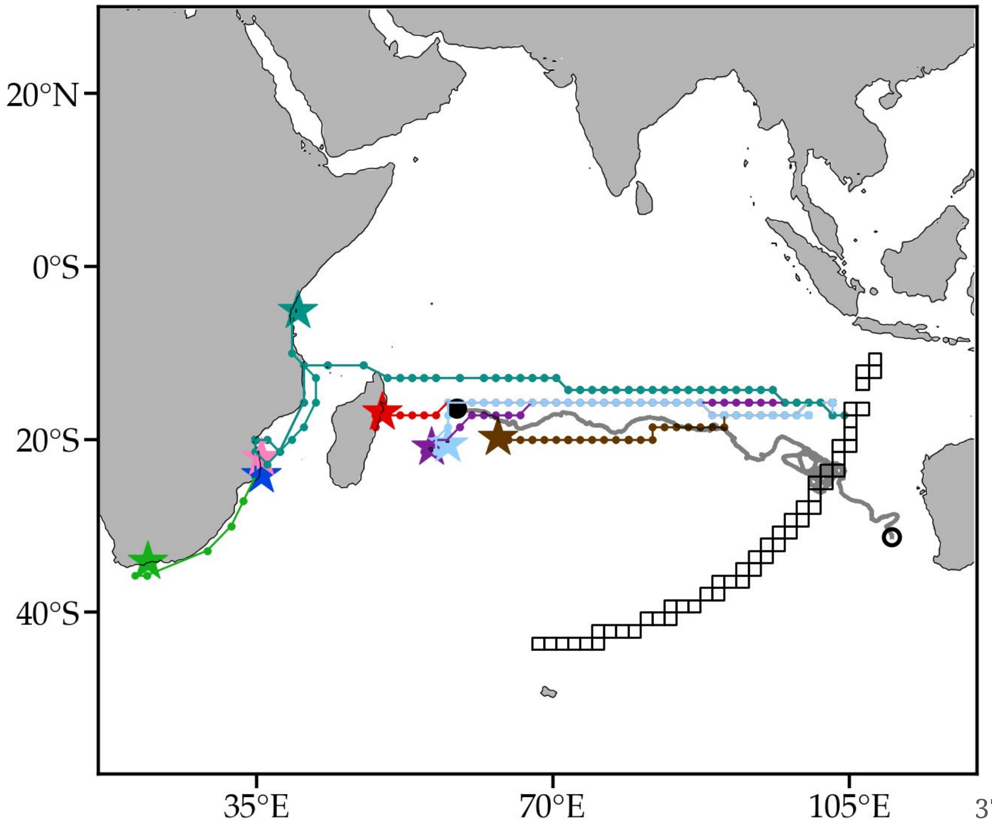


Most probable path

1. Each debris (location and endtime)
2. Pushforward from probable initial location *f(k=0)* until endtime
3. Each step and each bin *i*, store:
    a. The maximum probabilities (max $f(k-1)_j P_{ji}$)
    b. The bin *j* where this maximum probability comes from
4. On the last time step, same we have to reach the debris location.
5. Reconstruct the path backward

Most probable paths

- Closer to north group
- Not too far from a GDP drifters present at the plane disappearance (Trianes, 2016)
- North of the search area
- Individual path have very low probability of reaching the target at the right time



Future work

- *NSF project* : Nonlinear Dynamics Investigation of North Atlantic Deep Water Pathways (Beron-Vera, PI)
- Oil dispersion and evaporation (Olascoaga’s & Sheinbaum’s talk)